Span and Linearly Independent Sets of Vectors

1. What is the definition for a set of vectors $\vec{v}_1, ..., \vec{v}_n$ to span the entire $\mathbb{R}^m$ space they are in?

2. What is the definition for a set of vectors $\vec{v}_1, ..., \vec{v}_n$ to be linearly independent?

3. A set of vectors that span $\mathbb{R}^2$ but are not linearly independent?

4. A set of vectors that both span and are linearly independent in $\mathbb{R}^2$?

5. A set of vectors that are linearly independent in $\mathbb{R}^2$ but do not span $\mathbb{R}^2$?

6. A set of vectors that span $\mathbb{R}^3$ but are not linearly independent?

7. A set of vectors that both span and are linearly independent in $\mathbb{R}^3$?

8. A set of vectors that are linearly independent in $\mathbb{R}^3$ but do not span $\mathbb{R}^3$?

9. Revisit 1.4 #33, where $A$ is a 4x3 matrix and $\vec{b}$ is a vector in $\mathbb{R}^4$ with one unique solution.
   (a) Does $A$ have a pivot in every column? Why or why not?

   (b) Does $A$ have a pivot in every row? Why or why not?

   (c) What does Gauss-Jordan (reduced row echelon) look like for the augmented matrix $[A|\vec{b}]$, with the $\vec{b}$ that gives a unique solution?

   (d) Does $[A|\vec{b'}]$ have a unique solution for every $\vec{b'}$ in $\mathbb{R}^4$?

   (e) What do the columns of $A$ span geometrically?

   (f) Are the columns of $A$ linearly independent?

   (g) What is the geometry of the intersection of the rows of $[A|\vec{b}]$?