Test 1: Selections from 1.1–1.5, 1.7, 2.1–2.3 and applications

- 1.1, 1.2 & 1.5: Gaussian elimination, algebra and geometry of solutions of systems of equations...
- 1.4: connects everything together
- 1.3 and 1.7: algebra and geometry of vectors (linear combinations/mixing, span, li...)
- 2.1 and 2.2 : matrix algebra: $A + B$, cA, A^T, AB, $A^{-1}_{2\times 2}$, $\det(A_{2\times 2})$
- 2.3: theorem 8: what makes a matrix invertible [connects 2.2 to 1.1, 1.2, 1.3 and 1.7] & condition #, Hill cipher

Test 1: Selections from 1.1–1.5, 1.7, 2.1–2.3 and applications

- Study guide has sample partial test (and solutions)
  - Fill in the blank
  - Computations and Interpretations / Analyses
  - True/False Questions
- hw, problem sets, & clicker questions [solutions online]
- computations, definitions, critical reasoning & “big picture”

Handwritten reference card (I hand out) allowed as a way to include some important examples or concepts that you aren’t as comfortable with. You won’t have room for everything, and you should try to internalize as much as you can.

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$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} -2x_2 \\ x_2 \\ 1 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \text{line parallel to \_ through tip of \_}$$

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1.3 and 1.7: coefficient matrix vectors

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2.1, 2.2, 2.3: coefficient matrix

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1.3 and 1.7: coefficient matrix vectors do not span $\mathbb{R}^3$, not l.i., equals column is a linear combination (in the plane they span)

2.1, 2.2, 2.3: coefficient matrix is not invertible...

If A is invertible:
Multiply both sides by the inverse
Reorder parenthesis by associativity
Cancel A by its inverse
Identity reduces

Pivots:

$A_{n\times n}$ invertible $\rightsquigarrow$ I so full pivots & all of thm 8 works $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$A_{n\times n}$ not invertible $\rightsquigarrow$ row of 0s & all of thm 8 fails $\begin{bmatrix}1 & 4 & 7\\2 & 5 & 8\\3 & 6 & 9\end{bmatrix}$

A not square then no inverse, but can't negate other thm 8 statements $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$, $\begin{bmatrix}1 & 0\\0 & 1\\0 & 0\end{bmatrix}$

Topics to Study

- algebra and geometry of equations and their solutions; algebra and geometry of rows of a matrix
- Gaussian and Gauss-Jordan methods and history
- augmented matrix, coefficient matrix
- pivots/leading 1s
- unique, 0 or infinite solutions algebraically and geometrically
- parametrization of infinite solutions
- lines or planes intersecting in a point, line or plane according to the number of free variables in a parametrization
- homogeneous systems and their solutions
- equations meeting certain criteria and their solutions or consistency [like 2 equations in 3 unknowns, 3 equations in 2 unknowns...]
- algebra and geometry of vectors; algebra and geometry of columns of a matrix
- algebra and geometry of objects like Gaussian reductions like

t (column 1) + (column 2) [parallel to column 1 through the tip of column 2]
c(column 1) +d (column 2) [a plane through the origin if the columns are l.i. and a line through the origin otherwise, unless the columns were the trivial 0 vector].

- writing out the solutions of a system as a vector parametrization equation with homogeneous plus particular portions
- diagonal of a parallelogram, scaling along a line
- scalar multiplication, addition and matrix multiplication of matrices and vectors (and relationship to systems of equations)
- algebra and geometry of linear combinations and weights; mixing problems
- applications of linear combinations to manufacturing, physics...
- do we span R^2, R^3,... [setting up = general vector and obtaining never inconsistent, ie full row pivots]
- what do we span? [line, plane, hyperplane...]
- is a vector in the span?
- linearly independent [setting up = 0 vector and obtaining only trivial solution, ie full column pivots]
- span but not linearly independent; linearly independent but not span
- practical applications of span; of linearly independent
- Algebra of matrix multiplication [the definitions of AB, the number of multiplications in AB]
- Elementary Matrices like Matrix([[1, 0, 0], [0, 1, 0], [-2, 0, 1]]) representing the row operation r_3' = -2r_1 + r_3 or Matrix([[1,0],(-2,0]]) representing the row operation r_3' = -2r_1 + r_2
- Inverse of a Matrix as a concept $A.A^{-1}$=I and in Maple and the computational formula for the inverse of a 2x2 matrix
- Transpose of a Matrix
- Matrix algebra properties that do hold, often reasoned using some combination of

applying an inverse (if it exists) to both sides of an equation,
reorder parenthesis by associativity, cancel A by its inverse, and
reducing the identity to get whatever is left alone.
[like Ax=b has 1 unique solution $x=A^{-1}b$ when A^{-1} exists and x and b are the correct size column vectors...]
OR reasoned using arguments involving pivots/missing pivots

- Matrix algebra properties that don't hold and counterexamples [like 2 non-zero matrices that multiply to yield a zero matrix, AB is not necessarily BA...]
- Square matrix theorem (last slide of What Makes You Invertible): The following are equivalent for square matrices A_nxn:

A invertible, A reduces to the Identity
matrix, columns A span R^n, A has full row pivots,
columns A linearly independent, A has full column pivots,
Ax=b has 1 unique solution $x=A^{-1}b$,

- Negations of the square matrix theorem for non-invertible A_nxn matrices, like a square nxn matrix A that does not row reduce to the identity matrix is logically equivalent to the columns of A not spanning R^n.
- Examples and counterexamples of the square matrix theorem when A is not a square matrix [like examples of a matrix whose column vectors span but are not l.i...]
- Hill Cipher: coding using A.uncoded message and decoding using A^{-1}.coded message
- Condition Number [you do NOT need to know the formulas - just the big picture idea]
- Decimals versus fractions in a computer algebra software program like Maple

Some Maple Commands Here are some Maple commands you should be pretty familiar with by now for this test - i.e. I will at times show a command, and it may be with or without its output:

```
> with(LinearAlgebra): with(plots):
> A:=Matrix([[-1,2,1,-1],[2,4,-7,-8],[4,7,-3,3]]); ReducedRowEchelonForm(A);
```

1. If I use the implicitplot3d command in Maple on the equations corresponding to the rows of the augmented matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \end{bmatrix}$$

we would see 3 planes intersecting in a point

Adapted from Problem Set 1 # 1

2. $\begin{bmatrix} 1 & 4 & -2 \\ 0 & -12+h & 0 \end{bmatrix}$ is consistent ~~as long as h is not 12~~ for all h

Circle True OR (only if false) correct the statement after is consistent False - Adapted from 1.1 #21

- Study the homework questions and study guide, they help a lot.
- Do the assignments and problem sets. They help with everything. Do the practice tests fully.
- If you don't understand a topic, buckle down and review it and ask questions until it makes sense. Take advantage of all the resources available like office hours. Always review solutions online.
- Keep up with the definitions for the term sheets.
- Learn the 1st chapter stuff real good as it will keep coming back.