Linear Algebra: Sample Test 1 Questions on Selections from Chapters 1

Part 1: Fill in the Blank Questions (3 points each - 30 points total) There may be more than one possible answer for a fill-in-the-blank question. Full credit answers are ones that demonstrate deep understanding of linear algebra from class and homework.

1. In linear algebra, a vector means an ordered column representing magnitude and direction
   From 1.3 and the ASULearn glossary

2. An augmented matrix corresponding to three equations reduces to $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ The pivots are all three 1s
   Adapted from 1.2 Problem 3

3. What are the solution(s), if any, in #2? no solutions because 0x+0y=1 by row 3
   Adapted from 1.1 and 1.2 hw and clicker questions

4. Multiply $\begin{bmatrix} 5 & 8 \\ -2 & 3 \end{bmatrix}$ by-hand via $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (show work, but no need to reduce)
   $\begin{bmatrix} 5 \times -1 + 8 \times 1 \\ -2 \times -1 + 3 \times 1 \end{bmatrix}$ - can use the dot product method or the linear combination method.
   Adapted from 1.4 Problem 3

5. Adding two vectors $\vec{v}_1$ and $\vec{v}_2$ gives the diagonal of the parallelogram
   Adapted from 1.3 Problem 1

6. The row operation which turns $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix}$ to $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$ is (like $r_3' = -5r_1 + r_3$)
   $r_3' = -4r_1 + r_3$
   See 1.1 #31

7. If I use the implicitplot3d command in Maple on the equations corresponding to the rows of the augmented matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \end{bmatrix}$ we would see 3 planes intersecting in a point
   Adapted from Problem Set 1 # 1

8. We have repeatedly seen that we must be careful with Maple's linear algebra commands, because we can sometimes get incorrect answers. An example is when:
   we use decimals instead of fractions (or) we use ReducedRowEchelon on a matrix with unknowns in the array
   Adapted from Coffee mixing (or) Problem Set 1 hint sheet

9. In problem set 2, the center of gravity was an example of the linear algebra concept linear combination of vectors
   See problem Set 2 #1

10. If A is an $n \times n$ matrix, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then $A\vec{x} = \vec{b}$ has no solution(s). Adapted from a combination of Clicker questions 1.1 and 1.2 #4 and 1.4 classroom notes - it is 0 because it should be $n \times 1$ vectors to give 0, 1, or infinite solutions

Part 2: Computations and Interpretations (40 points)

There will be some by-hand computations and interpretations, like those you have had previously for homework, clicker questions and in the problem sets. You are not expected to remember page numbers or Theorem numbers, but you are expected to be comfortable with definitions, "big picture" ideas, computations, analyses...

You can expect this section to be a question with numerous parts, adapted from (or combining) questions like by-hand Gaussian of matrices: 1.2 #19, Problem Set 1 #1 or #2 and/or the algebra of vectors: 1.3 Problem #15, 1.4 Problem 13, 1.7 #9, Problem Set 2#2 or #3, for example.

Part 3: True/False (3.75 points each - 30 points total) Follow the directions below each:
Circle True OR correct the statements as directed:

a) The solution set of a linear system involving variables $x_1, ..., x_n$ is all lists of numbers $(s_1, ..., s_n)$ that makes each equation in the system a true statement when the values $s_1, ..., s_n$ are substituted for $x_1 ... x_n$, respectively.

Circle (True) OR (only if false) correct the statement after is. True - Adapted from 1.1 #23 to correct it

b) $\begin{bmatrix} 1 & 4 & -2 \\ 0 & -12+h & 0 \end{bmatrix}$ is consistent ~~as long as h is not 12~~ for all h

Circle True OR (only if false) correct the statement after is consistent

False - Adapted from 1.1 #21

c) The vector equation $x_1 \begin{bmatrix} 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$ is equivalent to the matrix equation $\begin{bmatrix} 5 & 1 & -3 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$

Circle True OR (only if false) correct the statement after equation. False - Adapted from 1.4 #9

d) The plane spanned by $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$ includes many vectors in that plane that are not on the same lines as the spanning vectors, such as $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

Circle (True) OR (only if false) correct the statement after such as.

This is the example I told you to memorize from class.

e) Two vectors that are linearly independent in $\mathbb{R}^2$ are ~~$S = \{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}\}$~~ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

False: Adapted from clicker question 1.3 # 2 and Problem Set 2 # 2.

f) The equation $\vec{x} = \vec{p} + t\vec{v}$ describes a line through $\vec{p}$ parallel to $\vec{v}$

Circle (True) OR (only if false) correct the statement after describes. Adapted from 1.5 #23 d to correct it

Circle True OR provide a counterexample:

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g) If one row in an echelon (Gaussian) form of an augmented matrix is [0 0 0 5 0] then the associated linear system is inconsistent.

Circle True OR provide a counterexample

$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \end{bmatrix}$

is a counterexample because this system has a unique solution

1.2 Problem 21 part e)

h) Any system of 3 linear equations in 2 unknowns is always inconsistent
Circle True OR provide a counterexample
$x + y = 0$
$x + 2y = 0$
$x + 3y = 0$
is a counterexample because $x = 0, y = 0$ is a solution
1.2 Problem 31