Linear Algebra: Sample Test 2 Questions

Part 1: Fill in the Blank Questions (3 points each - 30 points total) There may be more than one possible answer for a fill-in-the-blank question. Full credit answers are ones that demonstrate deep understanding of linear algebra from class and homework.

1. $\begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$ = (show work, but no need to reduce) $\begin{bmatrix} -1 \times 4 + 3 \times -2 & -1 \times -2 + 3 \times 3 \\ 2 \times 4 + 4 \times -2 & 2 \times -2 + 4 \times 3 \\ 5 \times 4 - 3 \times -2 & 5 \times -2 + -3 \times 3 \end{bmatrix}$

   See 2.1 number 5

2. The inverse of $\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$ is (show work, but no need to reduce) $\frac{1}{4 \cdot 3 - (-2)(-2)} \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

   See 2.2 number 1

3. To solve $A\vec{x} = \vec{b}$ with A as in the last question, we can solve $\vec{x} = A^{-1}\vec{b}$ (or reduce $[A|b]$)

   See 2.2 number 5

4. If the condition number is on the order of 10^4 then that tells us that we may lose up to 4 digits of accuracy

   Problem Set 3 #3 (it measures the asymptotically worst case of how much the solutions to a system of equations can change with small variations in the system.)

5. In linear algebra, span means set of all linear combinations

   From 1.3 and the ASULearn glossary (can be 1 or infinite solutions for any $\vec{b}$ in the span)

6. A rotation that rotates counterclockwise by θ is represented in matrix form as

   $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ Adapted from 1.8, 1.9 and 2.7

7. If I use the implicitplot3d command in Maple on the equations corresponding to the rows of the augmented matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \end{bmatrix}$ we would see 3 planes intersecting in a point

   Adapted from Problem Set 1 # 1c

8. $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ has columns that are not l.i. (or do not span or are missing a pivot)

   Adapted from 2.3 #3, class notes and theorem 8 [what makes a matrix invertible]

9. A real-life application of [a topic from the new material] is
   Hill cipher, Yoda, Digital animations/images (answer would depend on the topic chosen)

10. If A is an invertible $n \times n$ matrix, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then $A\vec{x} = \vec{b}$ has no solution(s). Adapted from a combination of Clicker questions in 2.2 #2 and Clicker questions in 2.1 #5 - it is 0 because it should be $n \times 1$ vectors to give 1 unique solution

Part 2: Computations and Interpretations (40 points)

There will be some by-hand computations and interpretations, like those you have had previously for homework, clicker questions and in the problem sets. You are not expected to remember page numbers or Theorem numbers, but you are expected to be comfortable with definitions, "big picture" ideas, computations, analyses...

You can expect this section to be a question with numerous parts, adapted from (or combining) these types of questions:

See solutions on ASULearn and be sure you could do similar problems

2.1 #9, 21, 23
Clicker in 2.1 and 2.2 #7
2.2 #13, 17, 21, 23
2.3 #19, 21, 23
Problem Set 3 #1 or 2 or 4
2.7 #9

Part 3: True/False (3.75 points each - 30 points total) Follow the directions below each:
Circle True OR correct the statements as directed:

a) If $A = \begin{bmatrix} 4 & 6 \\ 20 & 7 \end{bmatrix}$ then $\underline{5A =}$ $\begin{bmatrix} 20 & 6 \\ 20 & 7 \end{bmatrix}$ $\begin{bmatrix} 20 & 30 \\ 100 & 35 \end{bmatrix}$
Circle True OR (only if false) correct the statement after $\underline{5A =}$

Clicker questions in 2.1 #3

b) Each column of AB is a linear combination of the columns of ~~B~~ A using weights from the corresponding columns of ~~A~~ B.
Circle True OR (only if false) correct the statement after of

2.1 #15 b)

c) The transpose of a product of matrices equals the product of their transposes in the ~~same order~~ reverse order
Circle True OR (only if false) correct the statement after the 2.1 #15 e)

d) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .A$ is the same as modifying A via $r_2' = 3r_1 + r_2$

Circle (True) OR (only if false) correct the statement after via.

Clicker questions in 2.1

e) To keep a car on a curved race track, we can perform the appropriate matrix operations in the following order ~~(Rotate).(Translate_to_curve).car~~ (Translate_to_curve).(Rotate).car
Circle True OR (only if false) correct the statement after order False - Clicker questions in 2.7 #7

f) If the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then $A_{n\times n}$ has fewer than n pivot positions.

Circle (True) OR (only if false) correct the statement after then. 2.3 #11d)

Circle True OR provide a counterexample:

g) A not square can never have only the trivial solution.

Circle True OR provide a counterexample $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Clicker questions in 2.3 and Hill Cipher #2. False with l.i. columns.

h) If A is an nxn matrix then the equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbb{R}^n$.
Circle True OR provide a counterexample 2.3 #11c). False if A not invertible, with the appropriate $\vec{b}$ outside the span of the columns of A, like $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ & $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$