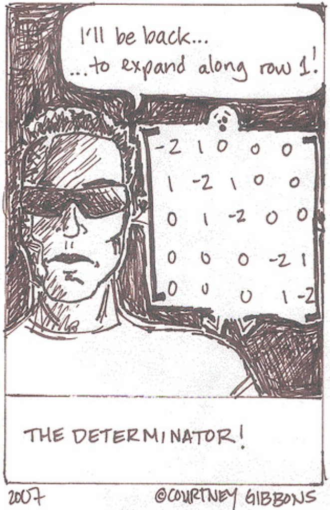
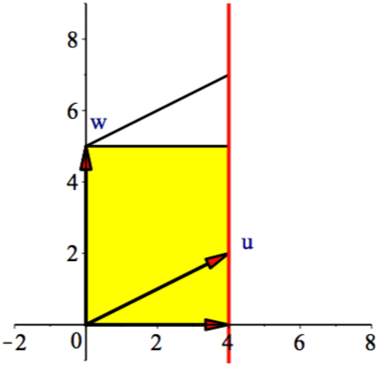
Test 3: + 2.8(146-150), 3.1(163-167), 3.2(169-172), 3.3(180-181), 5.1(265-269), 5.6(301-304) & apps

- Formatting same as prior tests. Test 3 is majority new material.
- hw, problem sets, & clicker questions [solutions online]
- computations, definitions, critical reasoning & “big picture”

- 1.1, 1.2 & 1.5: Gaussian elimination, algebra and geometry of solutions of systems of equations...
- 1.4: connects everything together
- 1.3 and 1.7: algebra and geometry of vectors (linear combinations/mixing, span, li...)
- 2.1 and 2.2: matrix algebra: $A+B$, cA, A^T, AB, $A^{-1}_{2\times 2}$, $\det(A_{2\times 2})$
- 2.3: theorem 8: what makes a matrix invertible [connects 2.2 to 1.1, 1.2, 1.3 and 1.7] & condition #
- 1.8 (62, 65, 67-68), 1.9 (70-75), 2.7: linear transformations
- apps: Hill cipher, computer graphics/animations, computer speed and reliability
- 3.1-3.3: algebra and geometry of determinants, invertibility
- 2.8: subspace, basis, column space and null space
- 5.1: eigenvectors & eigenvalues alg & geom, nullspace($A-\lambda I$)
- 5.6: eigenvector decomposition, limit, trajectory & populations



$$-2\cdot(-1)^{1+1}\begin{vmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{vmatrix} + 1\cdot(-1)^{1+2}\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{vmatrix} + 0s$$



Replacement $r_j' = -3r_1 + r_j$ are shear matrices when written in elementary matrix form like $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$—preserve the determinant (area, volume...) of a matrix and turns parallelograms and parallelopipeds formed by the vectors into rectangles and rectangular prisms

What Makes You Invertible

In verse

Music by One Direction & idea adapted from Art Benjamin
Interpreted by Dr. Sarah and Joel Landsberg

Baby you'll light up if one of these facts is so,
but you'll need n square columns and rows:

- Like when $\mathbb{R}^n$ is the span of the matrix columns
- That's when you know oh-oh invertible!
- If always you uniquely solve $A\vec{x}$ is $\vec{b}$
- Or if your columns have no linear dependency
- Or if matrix reduces to identity

Not zero - no no
That makes it not invertible!

Shout out if one of these facts is so...
but you'll need n square columns and rows:

- Like when your matrix determinant's non-zero

Is when you know oh-oh—that makes it invertible!

Subspace: it's linear (linear combinations)
Subspace basis: span (full row pivots) + l.i. (full column pivots)

reduced $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

column space of A=span of columns $[c_1 \vec{v}_1 + \ldots]$ or equations like $b_1 - 2b_2 + b_3 = 0$ from $[A|\vec{b}]$ reduction.
A basis is $\{$column 1, $\ldots$ of original matrix$\}$ - they span the column space via linear combinations and are l.i.

null space of A=solutions of $A\vec{x} = \vec{0}$. Aug: $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Parametrize: $s \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. A basis is $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

- $A\vec{x} = \lambda\vec{x}$: matrix multiplication to scalar multiplication
- determinant $(A - \lambda I) = 0$ or Maple for λ
- plug in each λ for nullspace of $(A - \lambda I)$ or use Maple for basis for eigenspace
- $A\vec{x} = \lambda\vec{x}$: A keeps eigenvectors $\vec{x}$ on the same line scaled by λ, so can reason geometrically for transformations

A := Matrix([[21/40,3/20],[19/240,39/40]]);

Eigenvectors(A); $\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} -6 & \frac{6}{19} \\ 1 & 1 \end{bmatrix}$

The eigenvector decomposition and trajectory:

$$\begin{bmatrix} \text{Owls}_k \\ \text{Rats}_k \end{bmatrix} = c_1 \left(\frac{1}{2}\right)^k \begin{bmatrix} -6 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} \frac{6}{19} \\ 1 \end{bmatrix}$$