Linear Algebra: Sample Test 3 Questions

Part 1: Fill in the Blank Questions (3 points each - 30 points total) There may be more than one possible answer for a fill-in-the-blank question. Full credit answers are ones that demonstrate deep understanding of linear algebra from class and homework.

1. The determinant of $\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ by-hand gives (show work, but no need to reduce) _____

2. A shear matrix is useful for _____

3. An elementary matrix that represents a shear matrix is _____

4. An eigenvector _____

5. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ has _____ real eigenvalues for most θ

6. A matrix that has all of $\mathbb{R}^2$ as its eigenspace is _____

7. If I use the implicitplot3d command in Maple on the equations corresponding to the rows of the augmented matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 5 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ we would see that the nullspace is a _____

8. A basis for the column space of $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ is _____

9. If A is an $n \times n$ matrix with a zero determinant, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then $A\vec{x} = \vec{0}$ has _____ solution(s).

10. If A is an $n \times n$ matrix with a non-zero determinant, and $\vec{x}$ and $\vec{b}$ are $1 \times n$ vectors, then $A\vec{x} = \vec{b}$ has ___ solution(s).

Part 2: Computations and Interpretations (40 points)

There will be some by-hand computations and interpretations, like those you have had previously for homework, clicker questions and in the problem sets. You are not expected to remember page numbers or Theorem numbers, but you are expected to be comfortable with definitions, "big picture" ideas, computations, analyses...

You can expect this section to be a question with numerous parts, adapted from (or combining) these questions:

3.1 #1
3.2 #42
3.3 #19, 25
2.8 #23
5.1 #2, 31
5.6 #3
Problem Set 4 #2, 3 or 4

Part 3: True/False (3.75 points each - 30 points total) Follow the directions below each:
Circle True OR correct the statements as directed:

a) det AB $\equiv$ detA det B
Circle True OR (only if false) correct the statement after $\equiv$

b) The volume of the parallelopiped formed by the column vectors of a matrix that is not invertible is 0.
Circle True OR (only if false) correct the statement after is

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & k & 1 \end{bmatrix}$ is not invertible
Circle True OR (only if false) correct the statement after is

d) The column space of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ is a subspace of $\mathbb{R}^3$
Circle True OR (only if false) correct the statement after of

e) If the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then the nullspace of A is at least a line
Circle True; OR (only if false) correct the statement after then.

f) To find the eigenvalues of A, solve by reducing A to echelon form
Circle True OR (only if false) correct the statement after solve by

Circle True OR provide a counterexample as directed:

g) If A is a 2×2 matrix then A must have 2 linearly independent (real) eigenvectors
Circle True OR provide a counterexample

h) If the largest eigenvalue equals 1, then the trajectory diagram would always have the populations dying off along that eigenvector.
Circle True OR provide a counterexample