The following are equivalent (TFAE) for linear independence (l.i.):

a. $c_1\vec{v}_1 + ... + c_n\vec{v}_n = \vec{0}$ has only the trivial solution $\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$ (i.e. the def of $\{v_1, ..., v_n\}$ l.i.)

b. $\begin{bmatrix} \vec{v}_1 & ... & \vec{v}_n \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$ has only the trivial solution $\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$

c. $\begin{bmatrix} & & & 0 \\ \vec{v}_1 & ... & \vec{v}_n & \vdots \\ & & & 0 \end{bmatrix}$ reduces to a matrix with a pivot position in every column except the = column

Compare to Theorem 4 for span where $\left[A|\vec{b}\right]$ reduces to a matrix that has no row $[0 \ldots 0|b_i]$, i.e. there is a pivot position in every row of A