I am feeling comfortable with the by-hand method of Gaussian/Echelon form with row reductions being used to obtain 0s below the diagonal:

a) Definitely
b) Somewhat
c) Unsure
d) Somewhat not
e) What are row reductions?

Use x_1 term in eq1 to eliminate terms below it via $r_k' = cr_1 + r_k$

$$\begin{bmatrix} 2 & 2 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 7 \\ 1 & 0 & -1 & -1 \end{bmatrix}$$

Use x_1 term in eq1 to eliminate terms below it via $r'_k = cr_1 + r_k$

$$\begin{bmatrix} 2 & 2 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 7 \\ 1 & 0 & -1 & -1 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 7 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r'_2 = -r_1 + r_2}$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 3 & 4 & 5 & 7 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r'_3 = -3r_1 + r_3} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 4 & 8 & 10 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r'_4 = -2r_1 + r_4}$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 4 & 8 & 10 \\ 0 & 2 & 4 & 5 \end{bmatrix}$$

Use x_1 term in eq1 to eliminate terms below it via $r_k' = cr_1 + r_k$

$$\begin{bmatrix} 2 & 2 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 7 \\ 1 & 0 & -1 & -1 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 7 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r_2' = -r_1 + r_2}$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 3 & 4 & 5 & 7 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r_3' = -3r_1 + r_3} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 4 & 8 & 10 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r_4' = -2r_1 + r_4}$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 4 & 8 & 10 \\ 0 & 2 & 4 & 5 \end{bmatrix}$$

Use x_2 term in eq2 to eliminate terms below it via $r_k' = cr_2 + r_k$

$$\xrightarrow[r_4' = -r_2 + r_4]{r_3' = -2r_2 + r_3} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Use x_1 term in eq1 to eliminate terms below it via $r_k' = cr_1 + r_k$

$$\begin{bmatrix} 2 & 2 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 7 \\ 1 & 0 & -1 & -1 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 7 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r_2' = -r_1 + r_2}$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 3 & 4 & 5 & 7 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r_3' = -3r_1 + r_3} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 4 & 8 & 10 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r_4' = -2r_1 + r_4}$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 4 & 8 & 10 \\ 0 & 2 & 4 & 5 \end{bmatrix}$$

Use x_2 term in eq2 to eliminate terms below it via $r_k' = cr_2 + r_k$

$$\xrightarrow[r_4' = -r_2 + r_4]{r_3' = -2r_2 + r_3} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Critical Analysis: Any [0 0 0 . . . 0 nonzero] ? (inconsistent)?

Use x_1 term in eq1 to eliminate terms below it via $r_k' = cr_1 + r_k$

$$\begin{bmatrix} 2 & 2 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 7 \\ 1 & 0 & -1 & -1 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 7 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r_2'=-r_1+r_2}$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 3 & 4 & 5 & 7 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r_3'=-3r_1+r_3} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 4 & 8 & 10 \\ 2 & 2 & 2 & 3 \end{bmatrix} \xrightarrow{r_4'=-2r_1+r_4}$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 4 & 8 & 10 \\ 0 & 2 & 4 & 5 \end{bmatrix}$$

Use x_2 term in eq2 to eliminate terms below it via $r_k' = cr_2 + r_k$

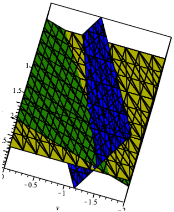
$$\xrightarrow[r_4'=-r_2+r_4]{r_3'=-2r_2+r_3} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

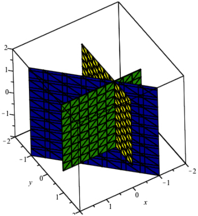
Critical Analysis: Any [0 0 0 ... 0 nonzero] ? (inconsistent)?

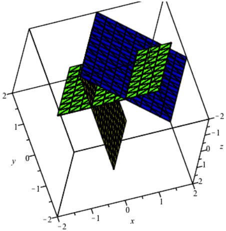
No. So solve via pivots (any variable missing a pivot is free).

pivots: 1 for x_1 & 2 for x_2, $x_3 = t$ Use row 2: $x_2 = \frac{5}{2} - 2t$, row 1: $x_1 = -1 + t$. Sols: 4 planes intersect in line $(-1 + t, \frac{5}{2} - 2t, t)$

How many solutions does each have?
Where in the room do we see each of these?







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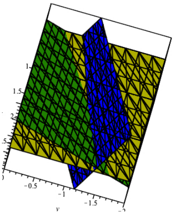

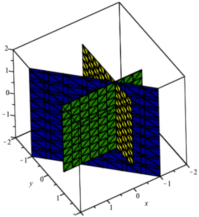

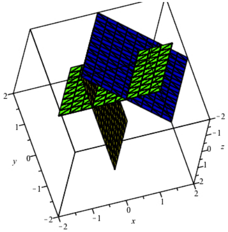

1 solution	infinite solutions	0 solutions
corner of room	book spine	hands + table

What does the Gaussian form look like in terms of pivots?

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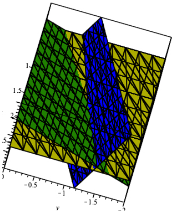

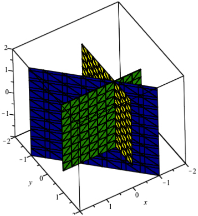

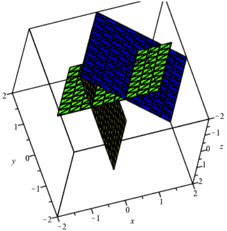

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What does the Gaussian form look like in terms of pivots?

full pivots   consistent & 1 variable w/ no pivot   [0 0 0 nonzero]

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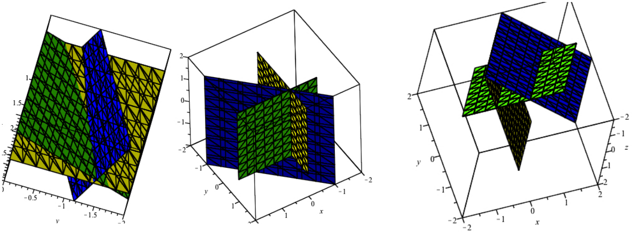

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What does the Gaussian form look like in terms of pivots?

full pivots   consistent & 1 variable w/ no pivot   [0 0 0 nonzero]

Note: consistent & only 2 variables w/ no pivots in $\mathbb{R}^3$ means a plane of solutions. Ex: $x + y + z = 1$. Augmented matrix: [1111] $y = s, z = t$, pivot for x gives $x = 1 - y - z$. Sols are plane $(1 - s - t, s, t)$