∞
Infinity

The history of infinity is a history infused with controversy. Consistently plagued with paradox, taboo, and even heresy, the chronicles of infinity are about as colorful and counterintuitive as a concept can get.

For millennia humanity has pondered the infinite and rigorously explored many methods for its apprehension. Originally operating strictly within the religious and philosophical realm, the idea of infinity began to emerge from the metaphysical abyss in 500 B.C.E. with the bizarre paradoxes put forth by Zeno of Elea. Zeno proposed over 40 paradoxes whose enigmatic and anomalous nature confounded philosophers and mathematicians for over 2000 years.

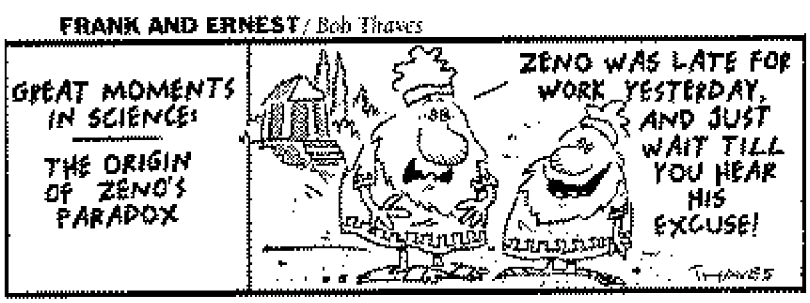
Zeno's progressive dichotomy paradox claims that in order for an individual to complete a movement from A to B, the individual must first make it to a halfway point C.

A -------------------------------C------------------------------- B

Upon reaching point C, the subject would now have a new distance, from C to B, to cover and would have to first make it to the halfway point D between them.

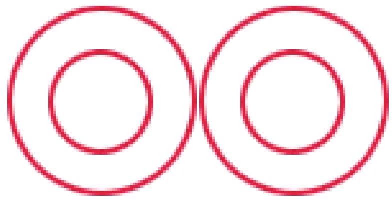
A --------------------------------C---------------D-------------- B

The numerical representation of this progression would be _+1/4+1/8+1/16+... This goes on *ad infinitum* and the subject never reaches their desired point. In fact, part two of this paradox, the regressive dichotomy, claims that motion is entirely impossible because if the above paradox is applied, the subject would not even be able to take the first step without first having to take a half step and then before that a quarter step and so on.



1) Using what you know about infinite series, explain Zeno's paradox.

In 1638, while writing *Discorsi e dimostrazioni matematiche intorno a due nuove scienze,* Galileo discovered an apparent contradiction with concentric circles.



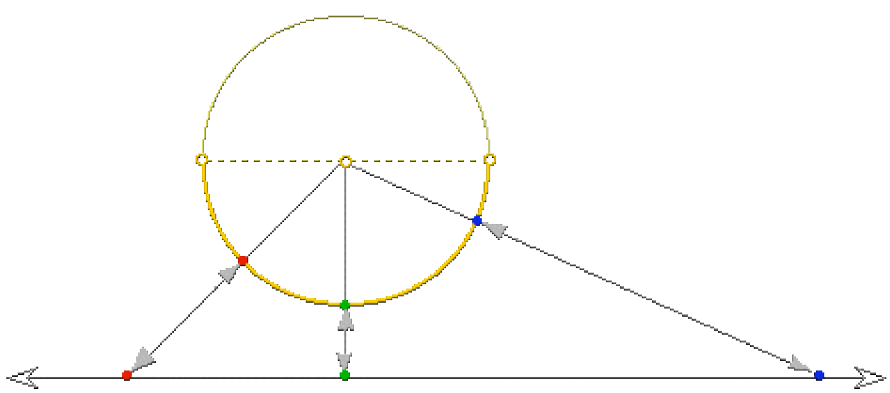
Although the outside circle has twice the diameter of the inside one, if you were to construct a radius of the larger circle and revolve it around both circles, every point on the larger circle would map one to one[1] with every point on the smaller circle!

Galileo then pointed out the following:

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & \dots & n & \dots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \\ 2 & 4 & 6 & 8 & 10 & \dots & 2n & \dots \end{array}$$

In this case, Galileo mapped on a one to one basis the set of natural numbers with the set of even natural numbers. Although the set of naturals intuitively appears twice as large as the set of evens, the mapping shows otherwise.

2) Use the drawing below to discern which figure, the finite semicircle or the infinite line, has more points. What if the endpoints on the semicircle were included? What about a full circle?



[1] A mapping which connects the members of one set E with members of another set M in such a way that a single element of M is associated with each element of E, and no two elements of E map onto the same element of M.

A more recent example of the counterintuitive nature of infinity is given by the following problem:

3) Consider the function $y=1/x$. Graph this function and then revolve the hyperbola around the x-axis. Using calculus, determine the surface area and the volume of the 3-dimensional figure. How does the surface area compare to the volume?

Worksheet Bibliography

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