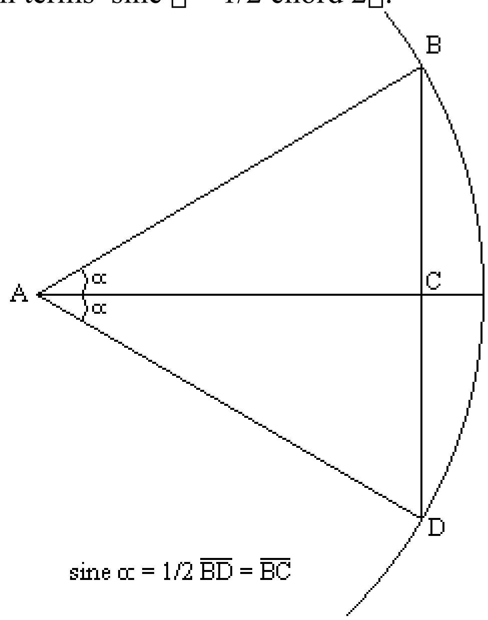
History of Logarithms

Napier (1550 - 1617): Napier created logarithms to reduce the amount of work it took to multiply two large numbers. Napier first referred to his logarithms as an "artificial number," but later he adopted the term logarithm. Logarithm translates as "the number of ratios" or "the reckoning number." Napier used his logarithms for calculating values of sine of an angle, in modern terms sine α = 1/2 chord 2α.



Napier calculated his logarithms using the series:

$$10^7(1 - 10^{-7})^n \text{ , } n = 0, 1, 2, \ldots , 100.$$

Note: the 10^7 come from the fact the Napier used a circle of radius 10^7.

Thus the Nap.log $10^7(1 - 10^{-7})^n = n$. Napier denoted his logarithms by adding a "Nap." to the beginning of the log, all that it meant was that this logarithm was calculated using his method, which was called a Napierian logarithm. This terminology can be some what confusing since natural logarithms are sometimes referred to as Napierian logarithms.

Calculate the value for x such that Nap.log x = 9.

Note: Napier did not really use a base since his logarithms were not exponents.
Today, some people only think of logarithms as some mathematical torturing device for

students; however, logarithms were actual very useful (and still are). To understand how logarithms were used, let's consider the product 15234 * 45219. This appears trivial with modern technology, but without a calculator this would take a moderate amount of time and the chances for human error are great. Now consider

$$\log(15234 * 45219) = \log(15234) + \log(45219)$$

Since these logarithms are to base 10 the product can be expressed as

$$10^{\log(15234)} + 10^{\log(45219)} = 15234 * 45219$$

Which further simplifies to

$$10^{\log(15234) + \log(45219)} = 15234 * 45219$$

Using the method shown above calculate the product of 823678 * 223747. You may only use your calculator to calculate the logarithm of the number and for raising 10 to the exponent. Show your work (**Note**: Given the limited number of decimal places for the logarithm your answer will be smaller than the actual product).

John Wallis: It is Wallis who is first accredited with making the connection between logarithms and exponents. It is this connection that gives logarithms so many of their useful properties, such as log (a * b) = log a + log b. It is easy to see that by raising both sides of the equation by 10, we get

$$10^{\log(a * b)} = 10^{(\log a + \log b)}.$$

Now using rules for exponents, separate the exponents on the right hand side of the equation, which yields

$$10^{\log(a * b)} = 10^{\log a} * 10^{\log b}$$

Now canceling the 10 with the logarithm yields

$$a * b = a * b$$

Using the same reasoning as above show that $\log(a^b) = b \log a$.

Logarithms have many interesting properties, but perhaps the most interesting is the fact that

$$d/dx \ln(x) = 1/x,$$

which was first discovered by Issac Newton in 1675.

To calculate the derivative of $g(x) = \ln(x)$ first let $f(x) = e^x$. Since $f(x)$ and $g(x)$ are inverses, then the following property is true $f`(g(x)) * g`(x) = 1$. Now solve this equation for $g`(x)$:

Next determine $f`(g(x))$ and substitute this into the equation $g`(x)$ to arrive at the desired results.