The fifth postulate and non-Euclidean Geometry

A timeline by Ederson Moreira dos Santos

300 B. C.	Euclid's Elements were published. The fifth postulate is not self-evident, and in virtue of its relative complexity and scant intuitive appeal, in the sense that it could be proved, many mathematicians tried to present a proof of it.
300 B.C	Aristoteles linked the problem of parallels lines to the question of the sum of the angles of a triangle in his Prior Analytics.
100 B.C	Diodorus, and Anthiniatus proved many different prepositions about the fifth postulate.
410-485	Proclus described an attempt to prove the parallel postulate due to Ptolemy. Ptolemy gave a false demonstration of if two parallel lines are cut by a transversal, then the interior angles on the same side add up to two right angles, which is an assertion equivalent to the fifth postulate. Proclus also gave his own (erroneous) proof, because he assumed that the distance between two parallel lines is bounded, which is also equivalent to the fifth postulate.
500's	Agh_n_s (Byzantine scholar) proved the existence of a quadrilateral with four right angles, which lead to a proof of the fifth postulate, and this was the key point of most medieval proofs of the parallel postulate, but his proof is based on an assertion equivalent to the fifth postulate.
900's	Ab_ 'Al_ ibn S_n_ (980-1037) defined parallel straight lines as equidistant lines, which is an equivalent proposition to the fifth postulate. The Egyptian physicist Abu 'Al_ Ibn al-Haytham (965-1041) presented a false proof for the fifth postulate, using the same idea of equidistant lines, to define parallel lines. The first attempt to prove the parallel postulate that is based on a more intuitive postulate is the theory of parallel lines of 'Umar Khayy_m, who used the idea of to construct a quadrilateral. This same idea was latter on (18^{th} century) used by Saccheri, and it has played a very important rule in the history of non-Euclidean geometry.
1600's	Pietro Antonio Cataldi (1548-1626), from Italy, assuming that there exist equidistant lines deduced a number of assertions from which it is already possible to prove the parallel postulate. John Wallis (1616-1703), English mathematician, also wrote a treatise dealing with the parallel postulate, where he proved the parallel postulate resting on the following postulate: to every figure, there exists a similar figure of arbitrary magnitude. Both assumption come to be equivalent to the fifth postulate.

1700's	Girolamo Saccheri (1667-1733), an Italian mathematician, made what turned out to be an important attempt to prove the parallel postulate, using a quadrilateral. In 1778 the Swiss mathematician Louis Bertrand (1731-1812) published a clever proof of the parallel postulate. But a mistake in his arguments was quickly brought to light by the Russian Emel'yanovi_ Gur'ev (1746-1813), and he, using another false conclusion, proved the parallel postulate.
1800's	In the first half of the 19th century there appeared several erroneous proofs of the parallel postulate by the Hungarian mathematician Farkas Bolyai. Friedrich Ludwig Watcher (1792-1817), a German student under Gauss, had attempted to prove the fifth postulate, and he believed that he had been successful. Bernhard Friedrich Thibaut (1775-1832) gave on other erroneous proof of the postulate. As we can notice above, during 2000 years, many mathematicians presented fake proofs, or proofs using assumptions that were equivalent to the fifth postulate, to prove it. Gauss was the first to have a clear view of a geometry independent of the fifth postulate (it was the birth of non-Euclidean geometry), and after him N.I. Lobatschewsky and Johann Bolyai. They share the honour of having made the first really systematic study of what we now call hyperbolic geometry. The existence of such non-Euclidean geometries proves that the fifth postulate is independent of the other ones, that is, it cannot be proved.
-	Ferdinand Karl Schweikark (1780-1859) developed a geometry independent of Euclid's hypothesis (fifth postulate).
1823	Lobatschewsky thought about Imaginary geometry. Bolyai discovered a formula that is the key for all Non-Euclidean trigonometries. The most interesting of the Non-Euclidean construction given by Bolyai is that for the squaring of the circle.
-	The full recognition that spherical geometry is itself a kind of Non-Euclidean geometry, without parallels, is due to Riemann (1826-1866).
1823-1860	The acceptance of the Non-Euclidean geometry was delayed by some reasons, such as the difficult of mastering Lobatschewsky's work written in Russian.
1860-1863	The correspondences between Gauss and Schumacher, published between 1860 and 1863, the numerous references to the works of Lobatschewsky and Bolyai, were a big step in direction to spread non-Euclidean geometries.
1871-1873	Klein suggested calling the geometries of Bolyai and Lobachewsky, Riemann, and Euclid, respectively, Hyperbolic, Elliptic, and parabolic.

1882	Henri Poincaré (1854-1912) presented two models, in a half-plane (which one I will present in my final project) and in a circle, for hyperbolic geometry.
1900's	Non-Euclidean geometries were widely spread, and the study of geometry on surfaces gave origin to differential geometry. Non-Euclidean geometry became a very useful tool for many areas, and a very wide field of research.

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