Trigonometry; Time to try it!

Introduction

Now that you have successfully completed College Algebra, it is time to move on to Trigonometry. Is this the part when you say "Ugh?" No, you can do it! You have made it this far! Contrary to what you may have heard from others, the concepts of trigonometry are not that hard, in fact, most of it is fun! Again, you can do it! So, what is Trigonometry and how exactly did trigonometry come to be a class you are now taking? Well, the history of trigonometry is actually quite long and convoluted because as with many areas in mathematics it wasn't the work of just one person but a major collaboration of many persons over many, many years. However, this worksheet will address a basic modern definition of trigonometry, some of the high points of the development of the topic, practical applications and give you a chance to try your hand at basic trigonometry. And you never know you just might grow to love it. Okay here we go!

A modern definition…

What exactly is Trigonometry? According to *The American Heritage Dictionary* the definition of Trigonometry is: ***The study of relationships between the sides and the angles of triangles.*** Cool! So we get to draw some triangles! But first, let us touch on a little history.

A brief history…

The history of Trigonometry has strong roots in the history of geometry. Both of which have been traced as far back as the Egyptian and Babylonian times. The Egyptians had a type of "pre-geometry" which is obvious in their pyramids and the Babylonians used trigonometry ideas to expand their interest in Astronomy. However, the bulk of the evolution of Trigonometry can be attributed to the Greeks.

So what about the terms *sine, cosine* and *tangent* that are used in trigonometry? How did they come about?

Sine: The first use of the idea of "sine" in the way we use it today was from the work *Aryabhatiya* of Aryabhata in 500AD. However, over the course of translating from the original language of Sanskrit, to Arabic, to Latin, and then to English there was a kind of translation "mix up." The concepts were clear, however the actual word was mistranslated. The resulting word *sinus* or *sine* was incorrect! It was suppose to be a different word!

Cosine: The origin of this *term* is a bit more modern. It came to be in 1620 when Edmund Gunter wrote *co.sinus* to denote the complementary angle of the sine. However, the actual concept of a cosine dates back to Egyptian times.

Tangent: The origin of this *term* is also modern. It was coined in 1583, by Dane Thomas Fincke. (A note for future math classes, *Tangent* is also used in Calculus and it has a related but somewhat different use and the concept has been around since the Greek times.)

Not just for class…

How is trigonometry used in everyday life? Numerous occupations require the use of trigonometry such as carpentry, architecture, surveying and basically any job in science or engineering. Trigonometry is not just some obscure form of mathematics that only looks good on paper, it has actual practical applications that you will see in one of the examples on this worksheet.

Working some problems…

So how do we get started using Trigonometry? First let us do a quick review of some common terms:

Hypotenuse: The longest side of a triangle.
Opposite: The far leg (or side) of the triangle.
Adjacent: The leg (or side) that touches the angle of interest.

How do we use sine (sin), cosine (cos) and tangent (tan)? Here are the relationships of the terms to the triangles that we will be discussing.

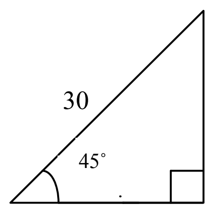
$$\sin A = \frac{opposite}{hypotenuse}$$

$$\cos A = \frac{adjacent}{hypotenuse}$$

$$\tan A = \frac{opposite}{adjacent}$$

Where *A* is an angle expressed in degrees.

Okay now on to the fun part! We get to draw! We will start with a basic right triangle. Recall that a right triangle has a 90° angle.



With the information given on the above triangle could you find the length of ***x***? Let's break it down. What do we have and what are we looking for? We know the length of the hypotenuse is 30 ft we also know that the one of the angles is 45°. What is ***x*** in relationship to the 45° angle? It would be the opposite side. So which of the trigonometric functions would we want to use? Which one has the two items that are given to us and the one we are looking for? Which one has a place to plug in a *degree value*? Answer: all three of them. Okay, which one has a place to plug in a *hypotenuse* value? Answer: Only the sine and cosine. Hey we have narrowed it down! And finally which one has a place to plug in a value (or in our case the variable ***x***) for the *opposite*? Answer: The sine! So plug them in and do some algebra!

$$\text{Sin } 45° = \frac{opposite}{hypotenuse} = \frac{x}{30}$$

$$\text{Sin } 45° = \frac{x}{30}$$

$$30(\sin 45°) = x$$

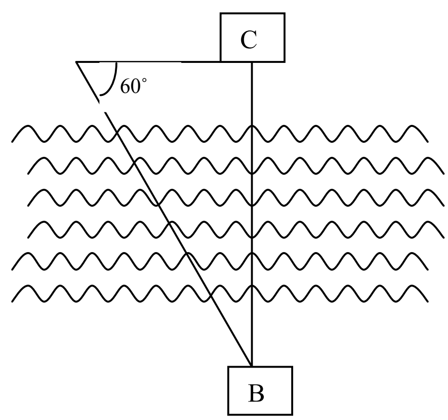
Now we have to either look up the value of the sine of 45° in a table or use a scientific calculator. (Note about the calculator, make sure you have the ***mode*** in ***degree*** not in ***radian***. Read your instruction manual or ask someone who is familiar with your type of calculator.)

Now back to the example…

The value of the sine of 45° is approximately .7071
So now just multiply
$30(.7071) = x$
$x = 21.213$
So what does that tell us? It tells us that the length of the side that is opposite to the 45° angle is approximately 21.213 ft long.

Okay ready to do another one? Here is an example that would be a good "real life application" of how one can use trigonometry in the work force.

It is your first day at ABC Surveying Company and you are being tested by your boss. Your challenge is to find the distance from cabin B to cabin C without traveling the distance across the river. You cannot use any of the company's equipment. You can only use your calculator, pen and paper. You have to use your knowledge of right angle trigonometry to solve the problem. Here is a diagram of the area with some additional information. Can you solve the problem? I bet you can! Let's do it together!



You would use the tangent function to find the opposite side.

$$\text{Tan } 60° = \frac{x}{330}$$

$$330(\text{Tan } 60°) = x$$

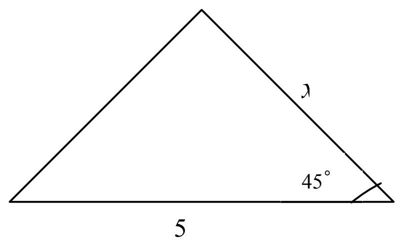
$$330(1.732) = x$$

$$x = 571.57 \text{ ft}$$

So it is approximately 571.6 feet from cabin B to cabin C. We did it! And without using expensive equipment or getting our feet wet!

Here are a few more problems for you to work on your own. The answers are at the end of the worksheet so you can check your work.

1)



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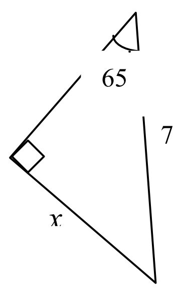

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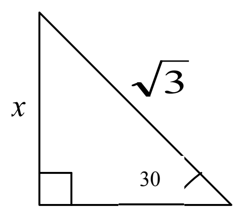

4) At a point 100 feet from away from the base of a giant redwood tree, a surveyor measures the angle of elevation to the top of the tree to be 70˚. How tall is the tree to the nearest tenth of a foot?

An additional challenge…

Now that you have had a chance to try your hand at right angle trigonometry, my challenge to you is to go on and explore more of what Trigonometry has to offer. You have your text book so *go ahead and read ahead*! Hey, why not check out the *unit circle*, it is a really neat concept and you will get to draw some more!

References

1) Joyce, David E. "Sines: The relation between sines and chords." Copyright 1999. Available:

http://aleph0.clarku.edu/~djoyce/java/trig/sines.html was used for the history of Sines.

2) Petterson, Jon Anders, Gjermund Vingerhagen, and Tommy Tveter Heggenes. "Trigonometry." Available: http://www-istp.gsfc.nasa.gov/stargaze/Strig2.htm
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3) http://pratt.edu/~arch543p/help/history_of_mathematics.html#1
Consulted for a general over all view of the historical material.

4) http://geometryalgorithms.com/history.htm Consulted for a general over all view of the history of geometry a precursor to trigonometry.

5) Boyer, Carl B. *A History of Mathematics.* 2nd ed. New York: Wiley and Sons, Inc., 1991. Consulted for historical information.

6) *The American Heritage Dictionary.* 1995. Consulted for a modern definition of Trigonometry.

7) Sobel, Max A. and Norbert Lerner. *Algebra and Trigonometry.* 4th ed. New Jersey: Prentice Hall, 1991. Consulted for examples and problems.

Answers: 1) □3.54 2) □6.06 3) □.866 4) □274.7 *ft*