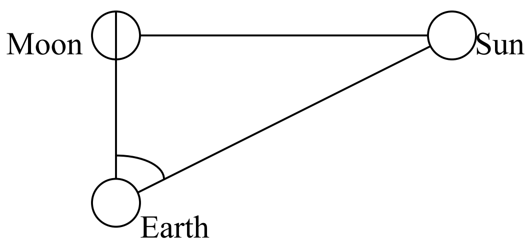
Krista Cornehl

Mathematics and Astronomy
Measuring Angles and the Relative Sizes of Planets

I. Measuring Angles -

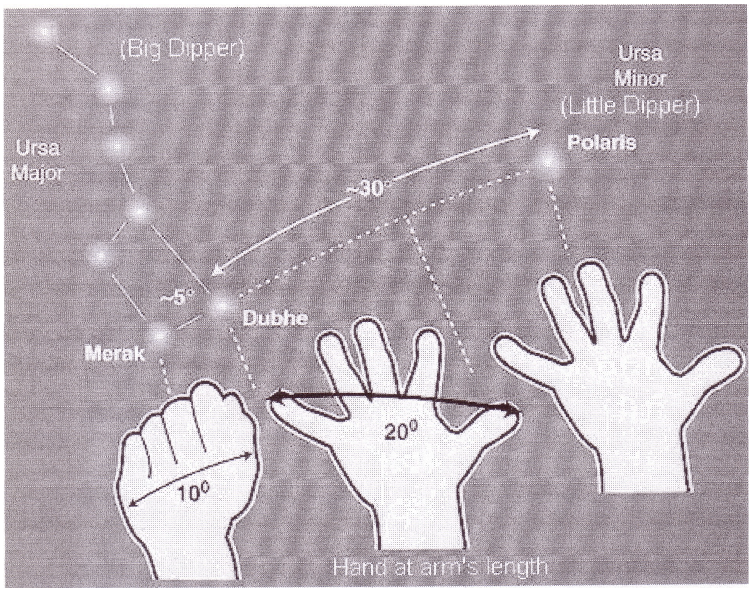
Part I. In ancient times the Greeks, Babylonians and Egyptians developed methods in order to approximate the distances between heavenly bodies, the angles between them, and the relative sizes of the bodies. Due to the fact that the telescope would not be invented for another 2000 or more years, these ancient civilizations had to rely on their eyesight for measuring and approximating these distances. Though the naked eye is not exact, there was a good sense of approximation. For example, Eratosthenes calculated the circumference of the Earth around 220 BC with only a 1% error of the actual value. Around 270 BC, Aristarchus developed a geometric method that allowed him to determine the ratio of the distances of the sun and moon from earth. He did this by measuring the angle between the sun and moon from earth during a day when the moon was only half illuminated by the sun (see the figure below).



Aristarchus used elementary trigonometry to calculate the ratio of the distances. Though his calculations were inaccurate, he did show the distance to the sun was much greater than the distance to the moon. This is the first attempt in history to approximate the dimensions of the solar system using actual observable measurements.

In order to measure angles, ancient civilizations had to use available materials (there were no telescopes or computers to make accurate measurements). Pretend that you are a person from an ancient civilization. You are summoned to measure angles between different celestial bodies, but your only tool is your hand.

Two different kinds of angle measurements can be taken. You can measure the angle from the earth's horizon to the heavenly body (this is called an altitude measurement). Or you can measure the angle between two heavenly bodies horizontally. For example, think about a compass. North is 90° from East. If you consider a triangle, you can measure an angle with the triangle standing upright (altitude) or the triangle lying on its side. Either way the triangle positioned you will get the same angle measure.



Picture taken from: http://www.geocities.com/angolano/astronomy/Piinsky.html

As illustrated in the picture to the left, your fist will be 10° and the length of your hand from the tip of you pinky to the tip of your thumb will be 20°. If you are measuring an angle greater than 20°, be sure to line your hands up end to end.

Directions: Go outside when it is dark and the stars are visible. Using your hand extended out fully in front of you, approximate the angles between the following:

1. The top of the handle of the Big Dipper and the North Star:______________________
2. Mars and the moon:_______________________
3. According to the diagram above, the top of the handle of the Big Dipper and the Dubhe:__________________________
4. The first and last star of Orion's belt:______________________
5. The angle from the earth to the moon:______________________
6. The angle from the earth to the top of the handle of the Big Dipper:_______________
7. Are these measurements accurate? Explain.__________________________________

____

____

8. Compare these measurements with another classmate's. Are they the same? If not, what are some reasons for why they are different?________________________________

____

____

<u>Part II.</u> The Greeks constructed a simple device called a Quadrant in order to measure angles the altitude (angle between earth's horizon and the object) of celestial bodies. Calculating the angles between planets at several times during the same night allowed people to hypothesize about the shape of the earth and the orbits of the earth and the planets. The Quadrant was improved over time. In the 1500s, Tycho Brahe invented a very accurate Quadrant allowing him to make a plethora of observations and measurements. He is known today for his tedious and meticulous observations that enabled him to show planetary positions as well as the development his own model of the solar system.

In the following activity you will construct your own Quadrant and use it to measure the angles of different celestial bodies.

Directions:
Step 1: See the attached page titled "Make a Quadrant." Follow the directions to make your own quadrant.
Step 2:
In order to use the Quadrant, align the ruler with the object you want to measure by sighting along the ruler. Make sure the ruler is directly pointing to the object that you are trying to measure. Let the weight hang down until it stops swinging. Rotate the ruler until the string is laying flat against the Quadrant. Hold the string in place and carefully read off the altitude (or vertical angle from you).

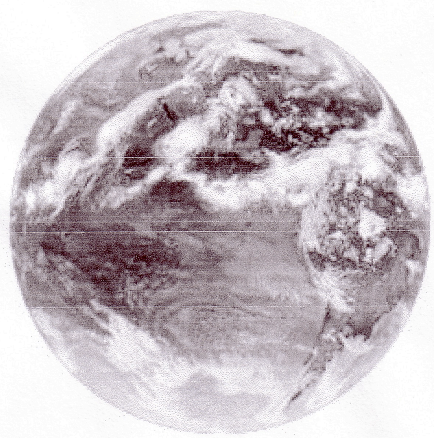
Step 3: Using the Quadrant calculate the altitudes of the following celestial bodies.
1. The sun (be careful not to look directly at it):________________________
The following measurement must be made **at night**.
2. The moon:______________________
3. The top of the handle of the Big Dipper:__________________________
4. Mars:___________________________
5. If you were to measure the altitudes of the Moon at different times of the night what do you think might happen?____
____
6. How do you think knowing this would help you devise a model for planetary motion?
____
____
7. Is a Quadrant more accurate than using your hand to measure angles? Explain._____
____
____
8. Compare your measurements with another classmate's. Are they the same? If not, explain why they might be different.____
____
____

II. Measuring Relative Sizes of Planets -

Knowing the altitude of a planet may help you get an idea of how large the planet actually is based on how large it looks to the naked eye and how far it is away. Models are useful in showing and explaining the ways in which the universe rotates and appears. Since we obviously cannot make a model as big as the universe, we need to make a scaled down model that is accurate. In order to make a smaller model, we need to start with an object that will represent one of the planets.

As mentioned earlier, Aristarchus found the ratio of the distance from the sun and moon to the earth. From this ratio he could come up with a scaled version of the solar system since each planet's distance from the earth is proportional to the ratio found. People studying astronomy needed a way to represent their observations and views of the solar system. In order to represent their system, they needed to draw or build a model. Once again, this drawing must be scaled down relative to a smaller object such as a coin. In 130 BC, Ptolemy wrote the *Almagest,* which described the positions, movements, and sizes of the planets. In his diagrams, Ptolemy had to fit the enormous size of the universe on a single sheet of paper. In order to accomplish this he had to find the sizes and distances of the planets relative to a smaller object.

For example: If the actual diameter of the Earth is 12,756.3 km, and we wanted to make a model of the planets based on an earth that was the size below, all we would need to do is a few conversions. Measure the diameter of the Earth below with a ruler and write it in the blank provided.



On the following page, you will complete an activity that uses the scaled diameter of the earth to scale the diameters of the rest of the planets.

Based on this information, find the scaled diameter of the following planets.
The following formula may be helpful to find the scaled diameter of each planet.

$$\frac{diameter\ of\ picture}{diameter\ of\ earth} \square diameter\ of\ planet = Scaled\ Diameter$$

Planet	**Actual Diameter**	**Scaled Diameter**
1. Mercury	4,880 km	
2. Venus	12,103.6 km	
3. Moon	3,476 km	
4. Mars	6,794 km	
5. Saturn	120,536 km	
6. Jupiter	142,984 km	
7. Neptune	49,532 km	

Picture and information from http://school.discovery.com/lessonplans/programs/scale/

*** Now complete this activity using the diameter of a penny as the scaled diameter of Mars.

Worksheet Goals and Objectives

The worksheet "Mathematics and Astronomy, Measuring Angles and the Relative Sizes of Planets" is intended for a high school audience. In order to complete this worksheet they must have knowledge of angles, measurements, ratios, and proportions.

Objectives:
- Understand different methods of measuring angles
- Understand accuracy of measurement and how to adjust for error
- Learn the significance of measurement and angles in history
- Understand how scaling factors can be used to make representations of astronomical distances
- Learn how to use conversions and ratios by writing and solving equations that relate real distance measurements to scaled representations of the distances

Materials:
- Hand
- Clear night sky
- Ruler
- String
- Weight
- Glue
- A piece of strong paper (cardboard or part of a folder)
- A Penny
- Pencil or Pen

References:
The history information came from the references in my project bibliography.

Specific information about the mathematicians and astronomers was also found at:
http://casswww.ucsd.edu/public/tutorial/History.html

The picture for measuring degrees with your hand:
http://www.geocities.com/angolano/astronomy/Piinsky.html

Information on the Quadrant:
Background information: http://www.aas.org/~aastra/quadrant.html
The picture and instructions: http://www.kyes-world.com/quadrant.html

Astronomical scale activity idea and pictures:
http://school.discovery.com/lessonplans/programs/scale/