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Mathematics and Astronomy
The Mathematics Involved in the Evolution of the Heliocentric Solar System

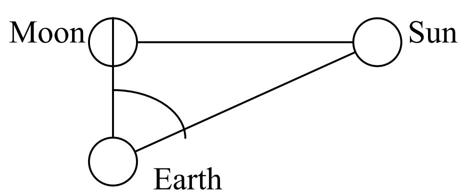
Astronomy has been a driving force in mathematics for thousands of years. Questions like: what is the distance between planets, how many planets are in the solar system, how large are the planets, what shape is the universe, how can planetary motion be described, and how far away from earth are the planets, are commonly asked. For thousands of years civilizations have struggled to answer questions like these, and the answers have been found by using mathematics.

Until the telescope was invented in the 1600s, people had to rely on their own vision to make observations of celestial bodies. As early as 1300 BC, the ancient Egyptians were using elementary geometry and astronomy to construct religious temples. For example, their pyramids were astronomically oriented (Neugebauer). Ancient Egyptians were primarily interested in astronomy because they needed to predict the annual flooding. They observed that Sirius (a star) rose just before the sun every 365 days, thus they developed a calendar based on 365 days (Boyer). Around 900 BC, the Babylonians developed a calendar based on astronomical data as well. They also divided the circle into 360 equal parts. The division of a circle into equal parts plays a crucial role in measuring angles in later civilizations (Kline).

About 600 BC, the Greek civilization began to make remarkable advances in astronomy using mathematics. The notion of a circular orbit was discovered from the Greek's knowledge of curved surfaces. The Greeks were also well aware of the fact that the earth is round, though around the 1500s people still thought it was flat ("Greek

Astronomy"). The first major astronomical contribution of the Greeks was made by Plato in 400 BC. He observed that celestial bodies are spherical and each has its own circular orbit. Around 360 BC, Eudoxus and Calippus were the first to attempt to represent celestial motion mathematically. Eudoxus represented the eight known heavenly bodies (Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn) as concentric spheres with the Earth at the center (measure un). He accounted for their motions using uniform circular motion (changing) (Boyer).

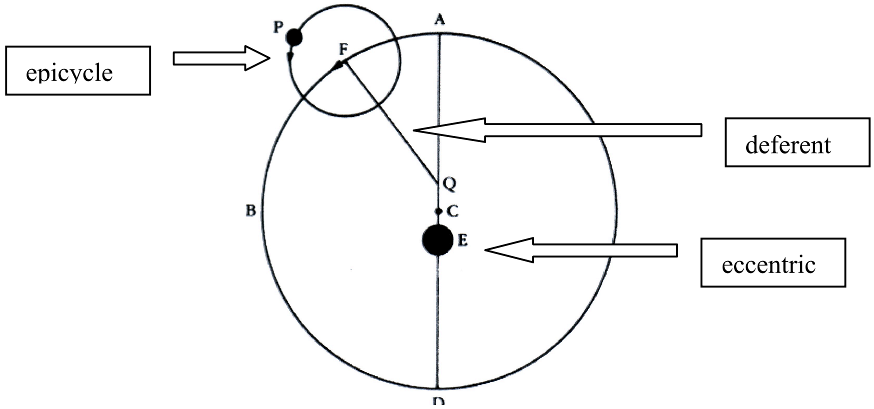
Aristarchus of Samos (250 BC) improved on Eudoxus' ideas. He was the first to attempt to measure distances of celestial bodies based on geometric principles (Boyer). Aristarchus was able to measure the relative distance of the sun and moon from Earth using angles. He knew that when the moon was exactly half illuminated by the sun that the $\angle$EMS was 90°. See the figure below.



Aristarchus measured $\angle$MES and recorded it to be 87° ("Aristarchus"). He probably measured this angle using a Quadrant or a similar device. A Quadrant is an instrument developed by the Greeks that measured astronomical angles (www.aas.org/~aastra/quadrant.html). Once these two angles were known, Aristarchus could find the ratio of the sides of the triangle. Using elementary trigonometry, he concluded that the ratio of the distance between the Earth and the sun to the distance between the Earth and the moon (ES:EM) was between 18:1 and 20:1. The actual value is 390, but this does show that Aristarchus' method is sound in principle (Maor).

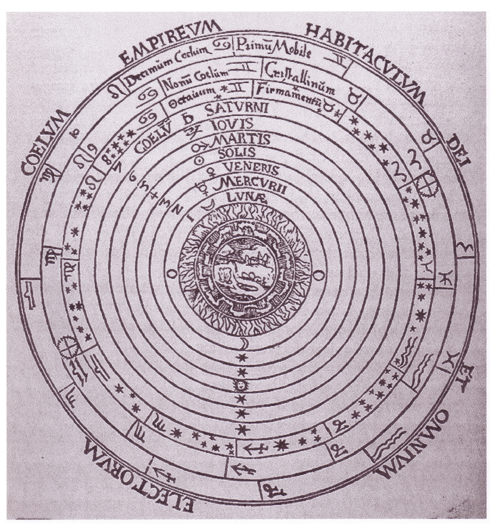
By 220 BC, a Greek named Eratosthenes calculated an approximation of the circumference of the Earth with only 1% error of the actual distance. He calculated this distance by using angles. On a day when the sun was directly above the town Syrene (in Egypt), it was 7.2° (1/50th of a circle) from directly above head in Alexandria (measured by driving a metal pole in the ground and measuring the shadow) (Maor). By assuming that the sun was a huge distance from the Earth, Eratosthenes figured the sun's rays reached the Earth parallel. Knowing the distance from Syrene to Alexandria was 5,000 stadia (a Greek measurement of distance), he found the circumference of the Earth to be 50 times this distance. This marks the first nearly accurate measurement of a portion of the solar system ("Eratosthenes of Cyrene").

Conic sections, elementary spherical trigonometry, chords, arcs, and circles were all developed and aided the study of astronomy. These topics of mathematics allowed famous solar system models such as the geocentric and heliocentric models to be developed. Ptolemy was a leading mathematician and astronomer in 130 BC. He developed the Ptolemaic Model (which is essentially the geocentric model) by building on Apollonius' previous ideas. His model was based on mathematical constructions that completely accounted for the motion of each celestial body ("Ptolemaic System"). Ptolemy had studied angular velocity as opposed to the previous linear velocity. He constructed his model using chords of arcs of circles and trigonometry. For example, he knew that $\sin^2 x + \cos^2 x = 1$. In his book the *Almagest*, he describes his improved model of celestial motion using epicycles, deferents, and eccentrics. An eccentric is a construction in which the Earth is placed slightly from the center of the universe. An epicycle is a circular orbit of a planet that also circles around the Earth; this accounts for the appearance of a planet to move forwards then backwards (retrograde motion). A deferent is the rotation of a planet around a fixed point (not the Earth or the center of the universe). See the model below for an illustration of the three different constructions.



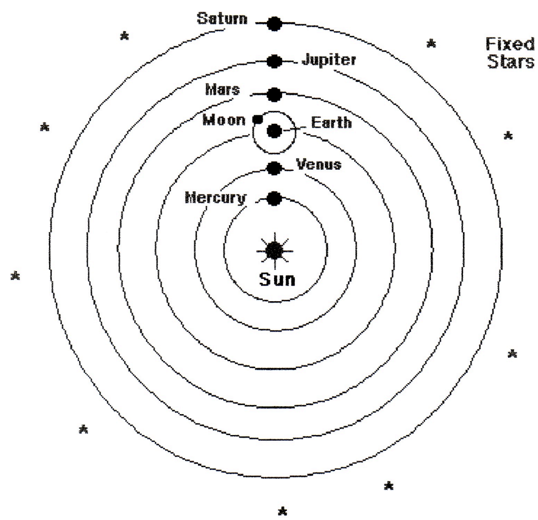
Picture from: es.rice.edu/ES/humsoc/Galileo/Images/Astro/Conceptions/combined_p.gif

His model was accurate according to the observations made during this time. Therefore, the Ptolemaic model was widely accepted since it explained all of the observable motions of the planets. The Ptolemaic model of the planets is shown below.



Picture from: www.seds.org/billa/psc/img/spheres.gif

After Ptolemy, a large period of time passed before any new discoveries were made in astronomy because during this time it was a popular belief that the Earth was flat. Not until the 1500s, when Copernicus developed his heliocentric model of the solar system, had any significant events occurred in astronomy. Copernicus founded the idea of the heliocentric solar system (one where the sun is the center of the universe and all of the planets revolve around the sun). He claimed the universe was spherical and extensively used trigonometric tables in his constructions ("Copernican System"). Copernicus' discoveries led to the use of calculus and Newton's gravitational laws. Below is a figure of his model.



Picture from: csep10.phys.utk.edu/astr161/lect/retrograde/Copernican.html

Originally, Copernicus' system was rejected by all, but his book *De Revolutionibus* was eventually used for the mathematical constructions it contained (Bell).

The Copernican system was improved in the 1600s by Kepler. He used a vast knowledge of ellipses to describe the elliptical orbits of the planets. Kepler also developed the idea of nonuniform velocities (Field). He is known for three discoveries in astronomy that are mathematically related. First, the orbits of planets were elliptical. He also discovered that "for each planet the line from the sun to the planet sweeps equal areas in equal times (Smith)." Finally, Kepler showed that the "square of the orbital period is proportional to the cube of the planet's distance from the sun (Smith)."

By the early 1600s the planetary model was developing into an accurate model of the universe as known today. With his invention of the telescope in 1630s, Galileo was able to convince the world of the heliocentric model of the solar system, and now accurate observations and measurements were easier to make (Ronan). By this time, there was also an idea of elementary Newtonian gravitation. Uranus was discovered by William and Caroline Herschel using a low powered telescope in 1781. Following this discovery, during the early 1800s Neptune had been observed numerous times through a telescope, but it was only thought of as a star. In the 1830s Adams and LeVerrier independently calculated that there must be another planet beyond Uranus using mathematics. (Due to observations of irregularities in Uranus' orbit, attention was brought to this area of the sky. Thus, mathematical calculations on these irregularities began and Adams and LeVerrier came up with their conclusion.) Neptune was then discovered purely by mathematical analysis. This marks the first time in history that a celestial body was discovered solely because of mathematics and not observation. The planet was first officially observed in 1846 by Hohan Galle ("Mathematical Discovery of Planets"). To complete the planetary model known today, Pluto was discovered in 1930 by Clyde Tombaugh.

As one can deduce, mathematics has been used for thousands of years to explain astronomical observations as well as predict them. Without mathematics, the motions and appearances of the universe could not be explained. From basic naked eye measurements and observations to intricate mathematical formulas and powerful telescopes, the heliocentric model of the universe is still developing into a more accurate model. As technology improves, so will our knowledge of the universe.

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Annotated Bibliography

"Aristarchus (310-230 BC)." Internet. 2 May 2003. Available: http://astrosun.tn.cornell.edu/courses/astro201/aristarchus.html.

This source shows how Aristarchus figured out the distances and sizes of the sun and moon.

Bell, E. T. (1937). Men of Mathematics. New York: Simon and Schuster.

This book emphasized the mathematics behind planetary motion and its characteristics. The main mathematicians involved with the mathematics behind the planetary and universe models and their work is included.

Boyer, C. B. (1991). A History of Mathematics (2nd Ed.). New York: John Wiley & Sons, Inc.

This book contains an enormous amount of material dealing with the mathematics behind astronomy, the changing views of astronomy (especially concerning the planetary model), and all of the mathematics that led to the mathematics of astronomy.

"Copernican System." Internet. 2 May 2003. Available: http://es.rice.edu/ES/humsoc/Galileo//Things/Copernican_system.html.

How Copernicus came up with his version of the solar system, and what his system looked like is discussed in this article.

Dunham, W. (1990). Journey Through Genius. New York: John Wiley & Sons, Inc.

Discusses the major accomplishments in mathematics in relation to astronomy. This reference does not contain much detail about the mathematics or astronomical discoveries.

"Eratosthenes of Cyrene." Internet. 27 Apr. 2003. Available: http://share2.esd105.wednet.edu/jmcald/Aristarchus/eratosthenes.html.

This source shows how Eratosthenes came up with his estimation of the circumference of the earth.

Ferguson, K. (1999). Measuring the Universe. New York: Walker and Company.

The different models of the orbits of the planets are briefly discussed. Mainly this book focuses on the measurements of the distances between the planets and the sun, the measurements of the relative sizes of planets as well as other objects in the universe, and how these measurements were made.

Field, J. V. (1988). Kepler's Geometrical Cosmology. Chicago: The University of Chicago Press.

This book intricately describes Kepler's planetary model. It includes detailed work of his as well as original drawings and sketches of his discoveries.

Gingerich, O. (1992). The Great Copernicus Chase and Other Adventures in Astronomical History. Cambridge: Sky Publishing Corporation.

This book gives a narrow overview of the advances in astronomy over the ages with general mathematical descriptions.

"Greek Astronomy." Internet. 24 Apr. 2003. Available: http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Greek_astronomy.html

This resource lists the Greek astronomers who made a contribution to astronomy, and briefly discusses how they came about this discovery.

Kline, M. (1972). Mathematical Thought from Ancient to Modern Times. New York: Oxford University Press.

This reference discusses the detailed mathematics behind astronomy. The Greeks and Pythagoreans are widely examined. This text also presents more modern mathematics concerning the planetary model by introducing mathematicians such as Copernicus and Kepler.

Lightman, A. (1991). Ancient Light. Cambridge: Harvard University Press.

This book contains information regarding Buddhist and Hindu models and advances in astronomy. It also contains a small amount of facts dealing with other ancient astronomers and mathematicians.

Maor, E. (1998). Trigonometric Delights. Princeton: Princeton University Press.

This book thoroughly discusses measurement in regard with astronomy. It contains information about how the measurements were made, what the measurements are, and who obtained the measurements.

"Mathematical Discovery of Planets." Internet. 24 Apr. 2003. Available: http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Neptune_and_Pluto.html.

This brief overview discusses how Uranus, Neptune, and Pluto were discovered.

Neugebauer, O. (1957). The Exact Sciences in Antiquity (2nd ed.). Providence: Brown University Press.

This text discusses astronomical discoveries and mathematics from ancient times, particularly dealing with the Babylonians and the Egyptians. Mesopotamian mathematics are also presented in this book.

Newman, J. R. (1956). The World of Mathematics (Vol. 1). New York: Simon and Schuster.

This book contains a plethora of information regarding mathematics, astronomy, mathematicians, and astronomers. Original diagrams and other sketches are very helpful and useful in this book.

"Ptolemaic System." Internet. 27 Apr. 2003. Available: http://es.rice.edu/ES/humsoc/Galileo/Things/Ptolemaic_system.html.

This source discusses the Ptolemaic model, what it is and how it works.

Ronan, C. A. (1961). Changing Views of the Universe. New York: The Macmillan Company.

This book discusses more precisely the changing ideas of the planetary model, and includes many diagrams on how these models worked. However, this reference does not discuss the mathematics in great detail.

Smith, G. "A Brief History of Astronomy." San Diego: University of California, Center for Astrophysics and Space Sciences. Available: http://casswww.ucsd.edu/public/tutorial/History.html.

This resource has a great timeline that includes astronomers and what they contributed mathematically. It also contains links to more detailed discussions of the mathematics and models presented in the timeline.

Smith, S. M. (1996). Agnesi to Zeno. Llenad, G., & Mills C. (Eds.). Berkeley: Key Curriculum Press.

This book has many small articles that are for use as a reference in school. The characteristics of Mayan astronomy are discussed.

Turnbull, H. W. (1993). The Great Mathematicians. New York: Barnes and Noble Books.

This book contains a great timeline of many well known mathematicians. It also discusses some topics in mathematics and astronomy.