Worksheet/History

The beginnings of matrices and Gaussian elimination started through the study of systems of linear equations, which was introduced by the Babylonians. The Babylonians were very advanced in mathematics, they divided the day into 24 hours, each hour into 60 minutes, each minute into 60 seconds. This form of counting (base 60) has survived for 4000 years. To write 5h 25' 30", i.e. 5 hours, 25 minutes, 30 seconds is just to write the base 60 fraction, 5 25/60 30/3600 or as a base 10 fraction 5 4/10 2/100 5/1000 which we write as 5.425 in decimal notation.

They had tables of squares, square roots, cubes, cube roots, reciprocals, exponential functions, log functions..... They had knowledge of trigonometry, the Pythagorean theorem 1200 years before Pythagoreas did, and pi. They knew that certain equation solutions reduced to log tables based on a non repeating fraction that they approximated as 2.43 in base 60 (163/60 or 2.716666.. in base 10). This is the base to the natural logarithm "e". They reduced equations to the quadratic form and solved some polynomial equations to the eighth degree. Unlike the Greeks, to follow 1000 years later , the Babylonians thought in terms of algebra and trigonometry instead of geometry.

About 1700 years later the Chinese came up with a new method, which resembles Gaussian elimination. It wasn't for another 1500 years before matrices were used in Europe.

THE CHINESE METHOD: The Chinese would use this method to solve everyday occurrences. An example of this would be:

3 cows + 2 horses + 1 chicken make 39 *dou*
2 cows + 3 horses + 1 chicken make 34 *dou*
1 cow + 2 horses + 3 chickens make 26 *dou*

What is the volume, in *dou*, of each animal?
To solve this, the Chinese wrote 3 equations, letting x=volume of 1 cow, y=volume of 1 horse, and z=volume of 1 chicken.

$$3x+2y+1z=39$$
$$2x+3y+1z=34$$
$$1x+2y+3z=26$$

The equations were not written in this fashion, but the coefficients of the unknowns and the constants were represented by rods on a counting-board as the array. For example:

3	2	1
2	3	2
1	1	3
39	34	26

For simplicity, we are going to use the first system of equations because it is was we're used to seeing.

Next, they would form a matrix using the coefficients and constants from the three statements.

Row 1	3	2	1	39
Row 2	2	3	1	34
Row 3	1	2	3	26

Step 1: Convert the numbers in the first column to zero

First multiply each number in Row 1 by the number in the first column of Row 2, in this example the number is 2. Call this new calculation Row A. Second, multiply Row 2 by the number in the first column of Row 1, in this example the number is 5. Call this new calculation Row B. Now subtract the number in each column of Row A from Row B. Call this difference Row C.

Row B - Row A	6 -6	9 -4	3 -2	102 -78
Row C	0	5	1	24

Step 2: In the first step, you use Row 1 and 2, now in this step you will be using Row 1 and 3. Multiply Row 3 by the number in the first column of Row 1 to get Row D and then multiply Row 1 by the number in the first column of Row 3 to get Row E. Now subtract the number in each column of Row D from Row E and call this difference Row F.

Row E -Row D	3 -3	2 -6	1 -9	39 -78
Row F	0	-4	-8	-39

Step 3: Now that we have converted the numbers in the first column to zero, we need to convert the number in the second column as well.

First we have to multiply Row F by the number in the second column of Row C and we'll call this Row G. Next multiply Row C by the number in the second column in Row F, and we'll call it Row H. Then subtract Row H from Row G and call it Row J.

Row G -Row H	0 -0	-20 -(-20)	-4 -(-40)	-96 -(-195)
Row J	0	0	36	99

Step 4: Now, since the first two columns are zero we can find the value of the third variable.

0 cows + 0 horses +36 chickens = 99

$36z=99$

$z=2.75$

Step 5: Lastly, we have to substitute $z=2.75$ in equations 2 and 3.

Use Row F to obtain

0 cows -4 horses -8 chickens = -39

$-4y-8(2.75)=-39$

$y=4.25$

Use Row 1 to obtain

3cows + 2horses + 1chickens = 39

$3x+2(4.25)+1(2.75)=39$

$x=9.25$

This is the method that the Chinese used to solve everyday equations. Then 1500 years later a man by the name of Gauss came up with Gaussian elimination, which resembles everything that we just did, but contains a lot less confusion. This worksheet assumes that you have already done Gaussian elimination.

1) Solve the following system of equations using a) the Chinese Method and b) Gaussian elimination

$2x-z=4$

$-x+2y=3$

$3x+z=2$

$-2x+3z=1$

a)

b)

2) Do Gaussian elimination and the Chinese method yield the same answer? If not, explain why it doesn't.

3) Discuss the differences between the two methods.

It took 1500 years for the Europeans to pick up on the new way of solving linear equations, but once Gaussian elimination was discovered, the research began. Now, we have a new, quicker way of solving these equations, the computer. Since the 1960's, research work on the design of algorithms and systems for performing symbolic mathematics (computer algebra) was underway at institutions such as MIT. When the MAPLE project was conceived in November 1980, the primary goal was to design a computer algebra system, which would be accessible to large numbers of researchers in mathematics, engineering, and science, and to large numbers of students for educational purposes. One of the key ideas was to make space efficiency, as well as time efficiency, a fundamental criterion. In this worksheet we are going to use Maple to find a quicker way to evaluate a systems of linear equations. I have outlined the necessary steps in achieving this below.

MAPLE:

Parameters
A - a rectangular matrix
'r' - (optional) for returning the rank of A
'd' - (optional) for returning the determinant of A

To include the linear algebra package you must first insert

> with(linalg):

and it will output

Warning, the protected names norm and trace have been redefined and unprotected

- disregard what it outputs
- Next, you must assign a matrix
- 3,3 represents the size of the matrix (row by column)
- [then just enter all of the coefficients and numbers in the equation]
- lastly, it will output your matrix, check this with your matrix, it should be the same.

> A := matrix(3,3, [x,1,0,0,0,1,1,y,1]);

$$A := \begin{bmatrix} x & 1 & 0 \\ 0 & 0 & 1 \\ 1 & y & 1 \end{bmatrix}$$

Then to perform Gaussian elimination, you enter in the next command

> gausselim(A, 'r', 'd');

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[x      1      0 ]
[    y x - 1     ]
[0   -------    1]
[       x        ]
[0      0      1 ]
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if you would like to find the rank of the system of equations just type in the following

> r;

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and if you would like to find the determinant of the system of equation just type the following

> d;

-y x + 1

4) Use Maple to solve the following system of equations:

2x-z=4
-x+2y=3
3x+z=2
-2x+3z=1

5) Does this yield the same answer as the previous two methods? If not, explain why.

6) Do you think in the future we will find a more simple way to solve systems of linear equations? Explain why or why not.

References for my worksheet

1. The History of Mathematics: an introduction / by David Burton – 5th edition, The McGraw-Hill Companies Inc, 2003

2. Elementary Linear Algebra / by Larson and Edwards – 2nd edition, D.C. Heath and Company, 1991

3. Algebra Activities from Many Cultures / by Beatrice Lumpkin