How has the Use of Linear Algebra Changed Throughout The Years
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The beginning of linear algebra originates around 1900 BC with the Babylonians. The Babylonians were the first to study systems of linear equations. Linear algebra came about when they studied problems that led to simultaneous linear equations. The Babylonians were very advanced in mathematics, they divided the day into 24 hours, each hour into 60 minutes, each minute into 60 seconds. This form of counting (base 60) has survived for 4000 years. To write 5h 25' 30", i.e. 5 hours, 25 minutes, 30 seconds is just to write the base 60 fraction, 5 25/60 30/3600 or as a base 10 fraction 5 4/10 2/100 5/1000 which we write as 5.425 in decimal notation.

They had tables of squares, square roots, cubes, cube roots, reciprocals, exponential functions, log functions..... They had knowledge of trigonometry, the Pythagorean theorem 1200 years before Pythagoreas did, and pi. They knew that certain equation solutions reduced to log tables based on a non repeating fraction that they approximated as 2.43 in base 60 (163/60 or 2.716666.. in base 10). This is the base to the natural logarithm "e". They reduced equations to the quadratic form and solved some polynomial equations to the eighth degree. Unlike the Greeks, to follow 1000 years later , the Babylonians thought in terms of algebra and trigonometry instead of geometry.

It wasn't until about 200 BC that the Chinese came up with a new method in solving these systems of equations. They used a method a lot like what we refer as

Gaussian elimination. The Chinese would use this method to solve everyday situations.

For example:

 3 cows + 2 horses + 1 chicken make 39 *dou*
 2 cows + 3 horses + 1 chicken make 34 *dou*
 1 cow + 2 horses + 3 chickens make 26 *dou*

What is the volume, in *dou*, of each animal?

To solve this, the Chinese wrote 3 equations, letting x=volume of 1 cow, y=volume of 1

horse, and z=volume of 1 chicken.

$$3x+2y+1z=39$$
$$2x+3y+1z=34$$
$$1x+2y+3z=26$$

The equations were not written in this fashion, but the coefficients of the unknowns and

the constants were represented by rods on a counting-board as the array. For example:

3	2	1
2	3	2
1	1	3
39	34	26

For simplicity, we are going to use the first system of equations because it is what we are

used to seeing.

Next, they would form a matrix using the coefficients and constants from the three

statements.

Row 1	3	2	1	39
Row 2	2	3	1	34
Row 3	1	2	3	26

Step 1: Convert the numbers in the first column to zero

First multiply each number in Row 1 by the number in the first

column of Row 2; in this example the number is 2. Call this new

calculation Row A. Second, multiply Row 2 by the number in the

first column of Row 1; in this example the number is 5. Call this

new calculation Row B. Now subtract the number in each column

of Row A from Row B. Call this difference Row C.

Row B	6	9	3	102
- Row A	-6	-4	-2	-78
Row C	0	5	1	24

Step 2:   In the first step, you use Row 1 and 2, now in this step you will be

using Row 1 and 3. Multiply Row 3 by the number in the first

column of Row 1 to get Row D and then multiply Row 1 by the

number in the first column of Row 3 to get Row E. Now subtract

the number in each column of Row D from Row E and call this

difference Row F.

Row E	3	2	1	39
-Row D	-3	-6	-9	-78
Row F	0	-4	-8	-39

Step 3:   Now that we have converted the numbers in the first column to

zero, we need to convert the number in the second column as well.

First we have to multiply Row F by the number in the second

column of Row C and we'll call this Row G. Next multiply Row

C by the number in the second column in Row F, and we'll call it

Row H. Then subtract Row H from Row G and call it Row J.

Row G	0	-20	-4	-96
-Row H	-0	-(-20)	-(-40)	-(-195)
Row J	0	0	36	99

Step 4:   Now, since the first two columns are zero we can find the value of

the third variable.

0 cows + 0 horses +36 chickens = 99
$$36z=99$$
$$z=2.75$$

Step 5:   Lastly, we have to substitute z=2.75 in equations 2 and

3.

Use Row F to obtain

0 cows -4 horses -8 chickens = -39
$$-4y-8(2.75)=-39$$
$$y=4.25$$

Use Row 1 to obtain

3cows + 2horses + 1chickens = 39
$$3x+2(4.25)+1(2.75)=39$$
$$x=9.25$$

It wasn't for another 1700 years that the next development was made in linear

algebra. Cardan made a rule for solving a system of two linear equations. This was the

first rule, concrete development, in linear algebra. Following this discovery,

mathematicians began to study these systems of equations and develop new rules.

The next major discovery was Cramer's Rule in 1750 AD. Cramer's rule is a

formula for solving systems of equations by determinants. Cramer's rule states that the

solutions for ax + cy =e and bx + dy =f are given by the determinant solutions.

$$x = \frac{\begin{vmatrix} e & c \\ f & d \end{vmatrix}}{\begin{vmatrix} a & c \\ b & d \end{vmatrix}} \quad \dots \quad y = \frac{\begin{vmatrix} a & e \\ b & f \end{vmatrix}}{\begin{vmatrix} a & c \\ b & d \end{vmatrix}}$$

When Cramer published his rule in 1750 he did not use determinants as they are now shown, and he gave no explanation for how he achieved the result. It is speculated that Colin Maclaurin probably discovered the same rule as early as 1729, but it was not published until after his death.

After Cramer's rule was published things began to pick up. Gauss first introduced the term determinant around 1800 AD, as well as matrix multiplication and the inverse matrix. Another major contribution was Gaussian elimination, in which I will explain: Suppose you need to find numbers x, y and z such that the following three equations are all true:

$2x + 4y + 2z = 15$
$2x + y + 2z = -5$
$4x + y - 2z = 0$

This is called a *system of linear equations* and the unknowns are x, y and z. The goal is to transform this system to an equivalent one so that we can easily read off the solution. The strategy is as follows: eliminate x from all but the first equation, eliminate y from all but the second equation, and then eliminate z from all but the third equation.

In this example, we eliminate x from the second equation by adding 3/2 times the first equation to the second, and then we eliminate x from the third equation by adding the first equation to the third. The result is:

$2x + y - z = 8$
$.5y + .5z = 1$
$2y + z = 5$

Now we eliminate y from the first equation by adding -2 times the second equation to the first, and then we eliminate y from the third equation by adding -4 times the second equation to the third:

$$2x \quad - 2z = 6$$
$$.5y + .5z = 1$$
$$\quad - z = 1$$

Finally, we eliminate z from the first equation by adding -2 times the third equation to the first, and then we eliminate z from the second equation by adding .5 times the third equation to the second:

$$2x \quad = 4$$
$$.5y \quad = 1.5$$
$$- z = 1$$

Now, we can easily read off the solution: $x = 2$, $y = 3$ and $z = -1$.

Sometimes it is necessary to switch two equations: for instance if y hadn't occurred in the second equation after our first step above, we would have switched the second and third equation and then eliminated y from the first equation. It is possible that the algorithm gets "stuck": for instance if y hadn't occurred in the second and the third equation after our first step above. In this case, the system doesn't have a unique solution, but I won't get into that in this paper.

Once this discovery was made, mathematicians really started getting into new ways of studying systems of equations. Vector spaces, matrix algebra and eigenvalues were soon discovered and the linear algebra we know today was formed.

Linear algebra today has been extended to consider n-space, since most of the useful results from 2 and 3-space can be extended to n-dimensional space. Although many people cannot easily visualize vectors in n-space, such vectors are useful in representing data. Since vectors are *ordered* lists of n components, most people can summarize and manipulate data efficiently in this framework. For example, in economics,

one can create and use, say, 8-dimensional vectors to represent the Gross National Product of 8 countries. One can decide to display the GNP of 8 countries for a particular year, where the countries' order is specified, for example, (United States, United Kingdom, France, Germany, Spain, India, Japan, Australia), by using a vector (v1, v2, v3, v4, v5, v6, v7, v8) where each country's GNP is in its respective position.

Linear algebra also plays an important part in analysis, notably, in the description of higher order derivatives in vector analysis and the study of tensor products and alternating maps. A vector space is defined over a field of real or complex numbers. Last but not least, linear algebra is important in computers and in this day in age, it's what we use the most. It is used in computer games, and everyday programs that we run.

In conclusion, over the years linear algebra really hasn't changed much. The Babylonians and the Chinese used it for everyday use, and in the 21st century, so do we.