Goals:

This worksheet is designed to help the student understand hyperbolic spaces as well as help the student understand mathematical models. In this worksheet, the student will prepare, via crocheting, a model of hyperbolic space. Listed below, in specific detail are the goals of this worksheet:

1. To understand hyperbolic space.
2. To understand the importance of mathematical models.
3. To learn to crochet.

There is no assumed background for this worksheet. Rather, this exercise is oriented to those students who have studied and also have an understanding for hyperbolic geometry. Generally however, these topics are covered at the collegiate level.
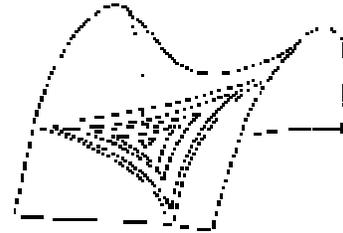
Materials:

There are only a couple materials needed for this exercise. These materials include:

1. A spindle of yarn.
2. A crochet needle (one).

Hyperbolic Visualization

Introduction

Hyperbolic space is a boundless infinite space with constant negative curvature in which any "slice" is a hyperbolic plane. The hyperbolic plane can be visualized in Euclidean space as a wavy sheet (as captured in the diagram to the right). If you were in this space, it would seem more confined than Euclidean space, because all of space would seem to be within a finite sphere. This misapprehension is caused by your vision, which is accustomed to Euclidean space and mistakes distances in hyperbolic space. This happens because the light from a distant object would enter your eyes at different angles. Furthermore, in hyperbolic geometry, lines, points, distance and motion all behave differently than their corresponding Euclidean counterparts. Therefore, we need models and graphs to ascertain universes such as hyperbolic space.

There are several models, which we could construct (the Poincare Disk, the Klein-Beltrami model, the Upper Half Plane, and Minkowski's model) to model hyperbolic geometry. However, we are going to concentrate on a crocheted model of the hyperbolic plane. The act of constructing the surface will give you a feel for hyperbolic geometry that is difficult to get any other way. In the next section of this worksheet, not only will you learn to crochet, but also you will construct, by means of crocheting, a model of the hyperbolic plane!

How to Crochet

1. The first step to crocheting is how to hold the hook:

   - Hold the hook between your thumb and index finger, like a pencil
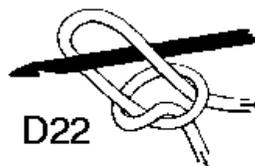
     (fig. 1).
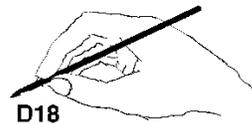
   - Hold the hook in an overhand manner, like a knife (fig. 2).

2. The next step to crocheting is to make the initial slip knot:

   - Make a pretzel shape with the yarn (fig. 3).

   - Slide the hook through the "pretzel" (fig. 4).

   - Pull on both ends of the yarn to tighten and adjust the size of the knot

3. Holding the yarn:

   - Yarn is to be held any way that is comfortable for you.

4. Making the chain:

- After making the slipknot, insert the hook, wrap the yarn around, hook, and pull back through the loop of the slipknot (fig. 5, 6).

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- Repeat the process as many times as you need chains (approximately 20 for this exercise).

5. Close off chain:

- Insert the hook into the first loop of the chain, and into the last loop in the chain, forming a circle (fig. 7).
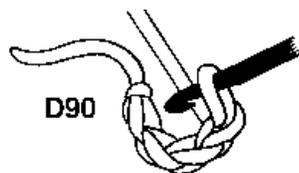
- Yarn over the hook, and bring the yarn back through the chain, this is also your first slipstitch. (fig. 8).


Now that you have created a chain, it is time to fill in the chain with stitches. The special way in which we crochet now will lead us to our anticipated hyperbolic model. The next set of instructions have accompanying figures which follow the current order and are also listed on the last page with the previous figures.

Crocheting the model

1. Stitching each row:

   - For the first stitch in each row, insert the hook into the second chain from the hook.
   - Take yarn over the hook and pull back through the chain (be sure to leave two loops on the hook).
   - Take the yarn over, again, and pull through both loops (fig. 9).
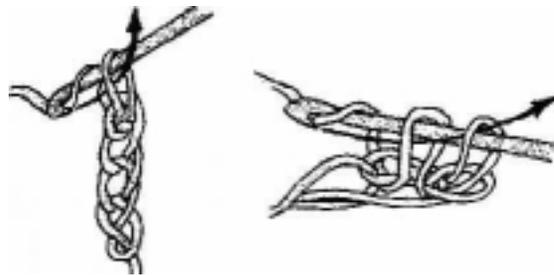
2. Stitching each row (continued):

   - For however many rows your creation is going to be, continue as before, but insert the hook into the next chain as oppose to the second chain.
   - Proceed as before except insert the hook into the last stitch to continue the row.
   - Repeat until you reach the end of the row.

3. At the end of the row:

   - Before going to the next row, you must also complete a chain stitch.

4. Closing off the creation:

   - Stop crocheting when you reach the last two loops.
   - Cut the yarn leaving approximately four inches.
   - Pull the yarn through the last two loops and tighten gently to prevent unraveling.

Finally, you will weave in the left over yarn. To do this, you will need a sewing needle. Thread the needle with the yarn and weave it in and out of the newly created object, until the loose end is no longer visible. Then, tighten the area where the loose thread was threaded into, to prevent it from unraveling. Once you have completed this step, your hyperbolic model should appear as below.

Utilization of the model:

In order to further understand hyperbolic geometry there are several experiments one can conduct. For example, fold the model as in the picture below. The line created by folding the model is where a "straight" line would lie on a hyperbolic plane. To us however, it appears a semicircle shape has been formed. To better understand this, imagine holding a bowl over your head,

looking up at the bottom. If the bowl goes on infinitely then a straight line will obtain curvature, and appear semicircular.

Another Example involves Euclid's fifth postulate which states, "Given a line I and a point *P*, not on the line I, there is exactly one line through *P* that is parallel to I". This postulate holds in Euclidean geometry. However, in hyperbolic geometry there is more than one line through *P* that is parallel to I, as shown below. This model causes Playfair's postulate not to hold in hyperbolic geometry.

In conclusion of this worksheet, it is a good idea to understand the topics covered thus far. Now that we know what the model helps us do, let's answer some questions about hyperbolic space:

1. What is Hyperbolic Space?

2. If we were in hyperbolic space, why would we feel more confined than in our normal world, where we are accustomed to Euclidean space?

References:

1. http://www.math.cornell.edu/~dwh/books/eg00/supplements/AHPmodel/index.html

This website contained the instructions for how to construct a hyperbolic model via crochet.

2. http://www.crochet.com/lessons/lessonr/rd2223.html

This website has the directions for crocheting

3. http://www.cs.brown.edu/people/tor/vr/doc/node5.html

This website contains information about hyperbolic space