Name: ______________________________
Date: ______________________________
Class: ______________________________

Why does every line have a perpendicular line to it?

Euclid, known as “the father of geometry”, is known for his book, Elements. In this book, he set up what we know today about modern Euclidean or plane geometry. He did this by defining several terms, having some basic assumptions known as postulates, and using common notations. From these three categories of information many propositions were and still are proved. The things that Euclid was able to prove are things we take for granted in Euclidean geometry. Some of ideas that Euclid had are unique to Euclidean geometry; it may not work in other geometries. In this exercise, we will prove one of his propositions of Book I using only what is allowed: a straight edge and a compass, as Euclid gives us:

The definitions of point, circle, center of a circle and the different types of triangles (equilateral, isosceles and scalene among others)

Postulates that say a line can be drawn from point to point, a circle has a center and a distance and all right angles are equal to each other.

Through previous propositions, SSS and SAS can be used and a line can be bisected.

We will prove the following proposition:

I-12: To a given infinite straight line, from a point which is not on it, to draw a perpendicular straight line. (Construct the perpendicular to a line from a point not on a line.)

Let’s see what we have: we have a line, a point not on the line, a straight edge and a compass.

We can’t just add a line and say it’s perpendicular. Since that eliminates the straight edge, let’s try the compass. We need another point to create a radius; let’s put it below the line. We need labels, so let’s label the original point A and the new point B. Why can we add another point? Because Euclid defines what a point is.

A .

B .

Draw in the circle with center A and radius AB . How many times does the circle intersect the line? Label those points the next letters in the alphabet.

Did we just create a line segment? Connect the ends of the line segment to the center of the circle. Are the new line segments you created equal? If so, why are they equal? Create a note of that on your drawing.

Is the triangle ACD equilateral? Why or why not?

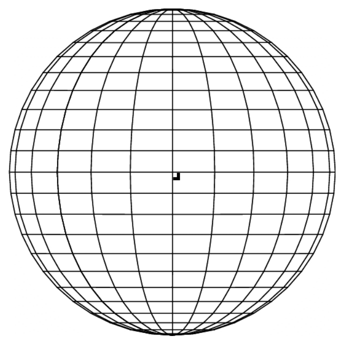
By one of Euclid's previous propositions, a line can be bisected. Bisect the line segment CD and call the point E on the drawing on the previous page. How are the line segments CE and ED related to each other in terms of length? Make a note of that on your drawing.

Create a new line segment from the point A to the point E. Can you see two triangles in your picture now?

Are triangles AEC and AED related to each other? How are they related to each other? SAS or SSS? (Euclid proves both of those.)

By SSS, angles CEA and AED are equal to each other. How many degrees are in the line segment CD? Then how many degrees are both in angles CEA and AED?

Did you create perpendicular lines? Which line segments are perpendicular to each other?
Perpendicular lines work will in Euclidean geometry and in real life. We see perpendicular lines in all aspects of life from building construction to baseball. Perpendicular lines work in other geometries, such as spherical geometry with the intersection of the equator and longitude lines such as seen below:



. It is something that many people do not understand, but now you do!

<u>Worksheet Goals:</u>

1. To introduce introductory geometry students to Euclid and his works
2. To introduce students to what a proof is.
3. To give students a small history of geometry.
4. To integrate many ideas of geometry into one proof, such as line, circle, SSS, etc.
5. To give students an exact idea of how two lines are perpendicular to each other.

References:

Euclid's <u>Elements</u> for the definitions, common notions, propositions and postulates.

Dr. Sarah Greenwald for the guidance in the proof and worksheet.