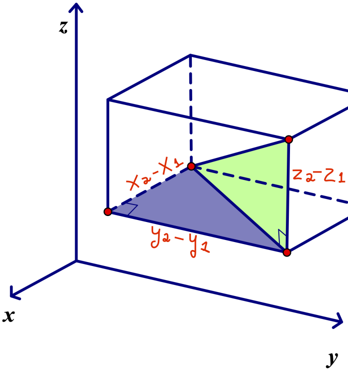
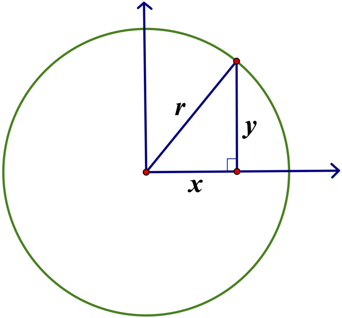
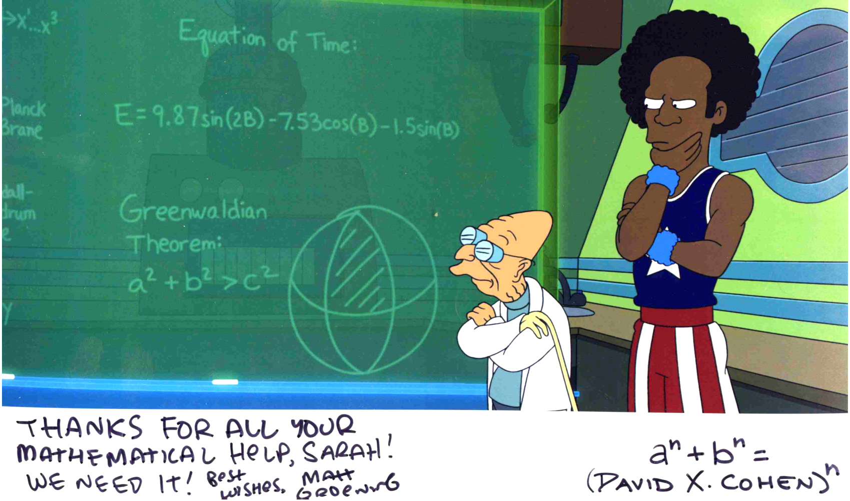
Analytic Geometry and Metric Perspectives





- Euclidean metric perspectives from Pythagorean theorem
- $d_{Euclidean}((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- $d_{Euclidean}((x_1, y_1, z_1), (x_2, y_2, z_2)) =$
  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- $ds^2 = dx^2 + dy^2$ in Calculus with Analytic Geometry
- circle: $r = \sqrt{x^2 + y^2}$ or $x^2 + y^2 = r^2$



$c_{\text{Euclidean}}$ is too long for sphere: $a^2 + b^2 = c^2_{\text{Euclidean}} > c^2_{\text{sphere}}$

(Poster made from *Bender's Big Score: Futurama*)

Taxicab Geometry Versus Euclidean Geometry

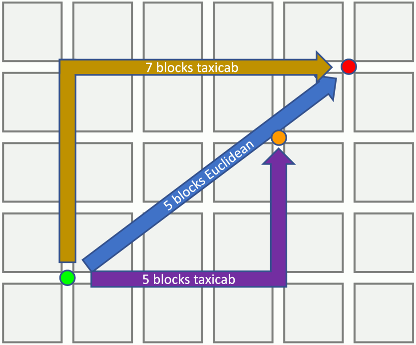


Image 1: part of Charlotte from Google Earth

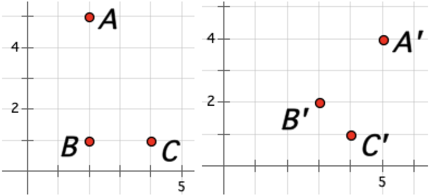
Image 2: `https://minecraft.fandom.com/wiki/Taxicab_distance`

$$d_{Euclidean}((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_{taxi}((x_1, y_1), (x_2, y_2)) = |x_2 - x_1| + |y_2 - y_1|$$

SAS in Taxicab Geometry

$A = (2,5), B = (2,1), C = (4,1)$
What is $\overline{AB}$, $\overline{BC}$, $\overline{A'B'}$, and $\overline{B'C'}$ in the taxicab metric?
$A' = (5,4), B' = (3,2), C' = (4,1)$



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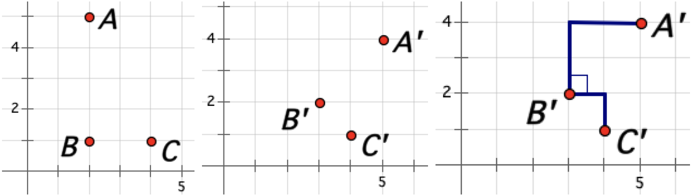


$d_{taxi}(A', B') = d_{taxi}((5,4),(3,2)) = |5-3| + |4-2| = 4$

To show that SAS fails in taxicab geometry, we will produce one counterexample that satisfies the assumptions of SAS but not the conclusion...

What goes wrong with the Euclidean proof of SAS?
`https://www.geogebra.org/geometry/enku7tgq`

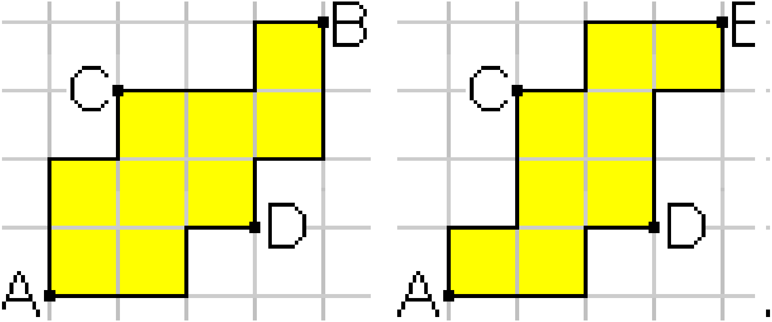
Proof by Counterexample

To show that SAS fails in taxicab geometry, we will produce one counterexample that satisfies the assumptions of SAS but not the conclusion. Look at $A = (2, 5), B = (2, 1), C = (4, 1)$ for $\triangle ABC$ and $A' = (5, 4), B' = (3, 2), C' = (4, 1)$ for $\triangle A'B'C'$. Now $d_{taxi}(A, B) = d_{taxi}((2, 5), (2, 1)) = |2 - 2| + |5 - 1| = 4$ and $d_{taxi}(A', B') = d_{taxi}((5, 4), (3, 2)) = |5 - 3| + |4 - 2| = 4$ so these sides are congruent. Also, $d_{taxi}(B, C) = d_{taxi}((2, 1), (4, 1)) = |4 - 2| + |1 - 1| = 2$ and $d_{taxi}(B', C') = d_{taxi}((3, 2), (4, 1)) = |3 - 4| + |2 - 1| = 2$ so these sides are also congruent. And, we can select taxicab paths so that the angles between these pairs of sides are 90 degrees. So the two triangles satisfy the assumptions of SAS. However, $d_{taxi}(A, C) = d_{taxi}((2, 5), (4, 1)) = |2 - 4| + |5 - 1| = 6$ and $d_{taxi}(A', C') = d_{taxi}((5, 4), (4, 1)) = |5 - 4| + |4 - 1| = 4$, which are not congruent. Thus SAS fails in taxicab geometry.

US law is Euclidean (16/01/2006)

This is what one James Robbins found out when he stood trial for drug dealing in New York last year. US law decrees that selling drugs within 1000 feet of a school carries extra penalties. Unfortunately, Robbins had been caught within that radius, so his lawyer decided simply to change the metric. He argued that the Euclidean metric, which measures distances along straight lines between points, should be replaced by a “Taxicab metric”, which measures distances along the roads a taxi, or a pedestrian, has to take to get from point to point. After all, students from the school in question are unlikely to walk through brick walls. According to this new metric, Robbins was 1254 feet away from the school: 764 feet north along Eighth Avenue and 490 feet west along 43rd Street. Alas, judge and jury were unimpressed by this mathematical trickery and Robbins lost.

Squares in Taxicab Geometry



http://www.mathematische-basteleien.de/taxigeometrie.htm

Euclidean Metric Applied to Spherical Surface Area

surface of revolution surface area = $\int 2\pi f \sqrt{1 + f'^2} dx$

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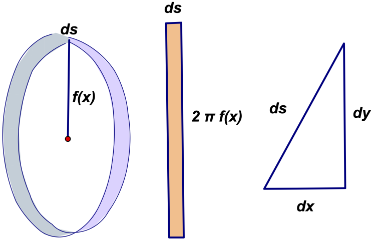

$$2\pi fds = 2\pi f\sqrt{dx^2+dy^2}$$

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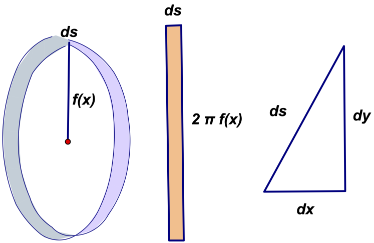

$$2\pi f ds = 2\pi f \sqrt{dx^2 + dy^2}$$

$$= 2\pi f \sqrt{dx^2 (1 + \frac{dy^2}{dx^2})} = 2\pi f \sqrt{1 + f'^2} dx$$

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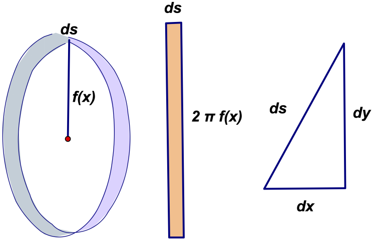

$2\pi f ds = 2\pi f\sqrt{dx^2+dy^2}$

$= 2\pi f\sqrt{dx^2(1+\frac{dy^2}{dx^2})} = 2\pi f\sqrt{1+f'^2}dx$

Here $f(x) = \sqrt{r^2-x^2}$ with r constant, so

$f'(x) =$

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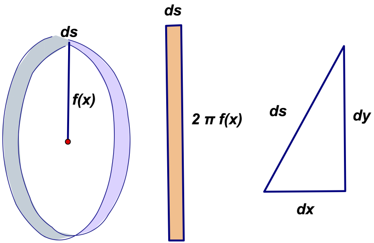

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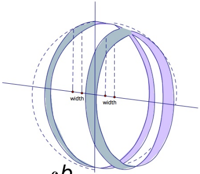
Here $f(x) = \sqrt{r^2 - x^2}$ with r constant, so

$f'(x) = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}(-2x) = -\frac{x}{\sqrt{r^2 - x^2}}$ and $f'^2 = \frac{x^2}{r^2 - x^2}$

Euclidean Metric Applied to Spherical Surface Area



spherical slice SA $= \int_a^b 2\pi f \sqrt{1+f'^2}dx$

$= \int_a^b 2\pi \sqrt{r^2 - x^2}\sqrt{1 + \frac{x^2}{r^2-x^2}}dx$

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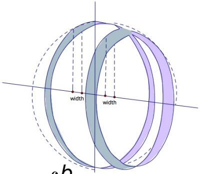

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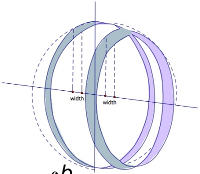

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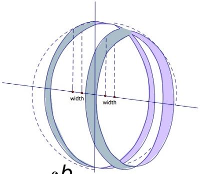

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$= \int_a^b 2\pi\sqrt{r^2-x^2}\frac{r}{\sqrt{r^2-x^2}}dx$

$= \int_a^b 2\pi r dx = 2\pi r\int_a^b dx = 2\pi x\Big|_a^b = 2\pi r(b-a)$

entire sphere: $2\pi r(r--r)$

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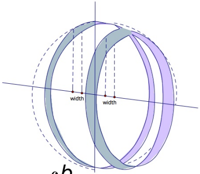

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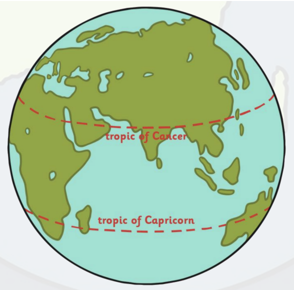
$$= \int_a^b 2\pi\sqrt{r^2-x^2}\frac{r}{\sqrt{r^2-x^2}}dx$$

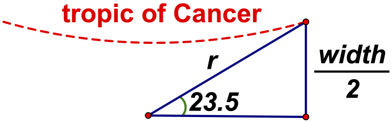
$$= \int_a^b 2\pi r dx = 2\pi r\int_a^b dx = 2\pi x\Big|_a^b = 2\pi r(b-a)$$

entire sphere: $2\pi r(r--r) = 2\pi r(2r) = 4\pi r^2$

What Fraction of the Earth is in the Tropics?

what fraction of the earth is in the tropics?
latitudes approximately +23.5 and -23.5 degrees





https://www.stillness-inf.lewisham.sch.uk/wp-content/uploads/2020/06/Y2-Geog-week-2.pdf

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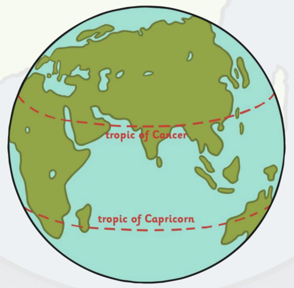

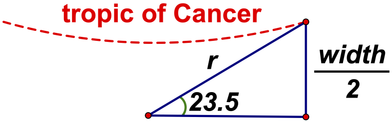

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$$\frac{2\pi r \text{width}}{4\pi r^2} = \frac{\text{width}}{2r}$$

$\sin 23.5^\circ = \frac{\text{width}}{2r}$ =fraction of the earth in tropics