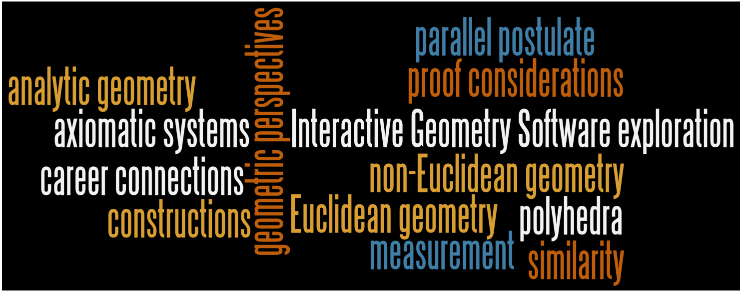
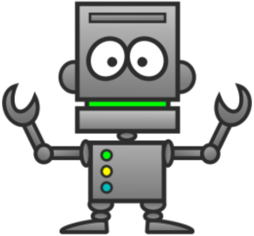
Axiomatic Systems and Constructions





Martin Berube

Peanut Butter and Jelly Robot

Axiomatic System: Minesweeper

- Axiom 1) Each square is a number or a mine.
- Axiom 2) A numbered square represents the number of neighboring mines in the blocks immediately above, below, left, right, or diagonally touching (or a subset of those if a block is on a boundary)

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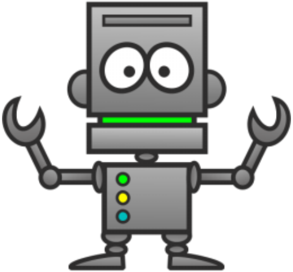
	A	B	C
1			1
2	2		*
3			1



http://img.gamefaqs.net/screens/a/1/8/gfs_45331_3_3.jpg

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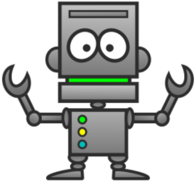
Proof: Given that C1=1, A2=2, C2=mine, and C3=1, to show that B1 is a number, assume for contradiction that it is a mine. Now this mine is adjacent to C1, which already has adjacent mine C2. Since C1 is a number, we know that it is at least a 2 by axiom 2, a contradiction to the fact that C1 is already a 1. So B1 is not a mine. Lastly, by Axiom 1, B1 must be a number, as desired.



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Axiomatic System: Incidence Geometry

- Axiom 1) For each two distinct points there exists a unique line on both of them.
- Axiom 2) For every line there exists at least two distinct points on it.
- Axiom 3) There exist at least three distinct points.
- Axiom 4) Not all points lie on the same line.

Proposition 1: There exist at least three distinct lines not through any common point.

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Proof. To prove Proposition 1, first, by Axioms 3 and 4, we can find A, B, and C as distinct points that are not collinear. By Axiom 1, we have unique lines $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$.

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David Henderson:

An ideal proof

is a communication—*when we prove something we are not done until we can communicate it to others and the nature of this communication, of course, depends on the community to which one is communicating and is thus in part a social phenomenon.*

is convincing—*a proof "works" when it convinces others. Of course some people become convinced too easily so we are more confident in the proof if it convinces someone who was originally a skeptic. Also, a proof that convinces me may not convince you or my students.*

answers–Why?– *The proof should explain something that the hearer of the proof wants to have explained. I think most people in mathematics have had the experience of logically following a proof step by step but are still dissatisfied because it did not answer questions of the sort: "Why is it true?" "Where did it come from?" "How did you see it?" "What does it mean?"*

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Consistent?

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Models?

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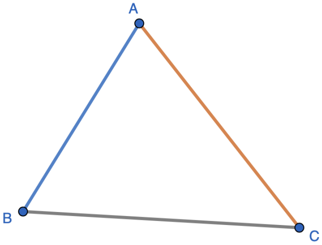

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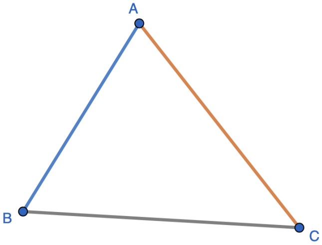

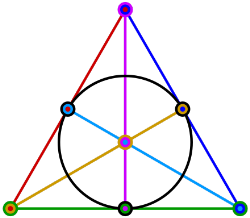

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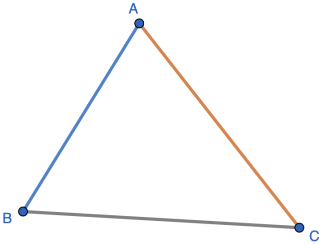
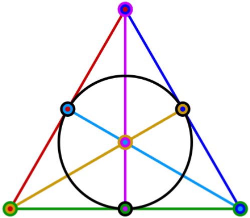
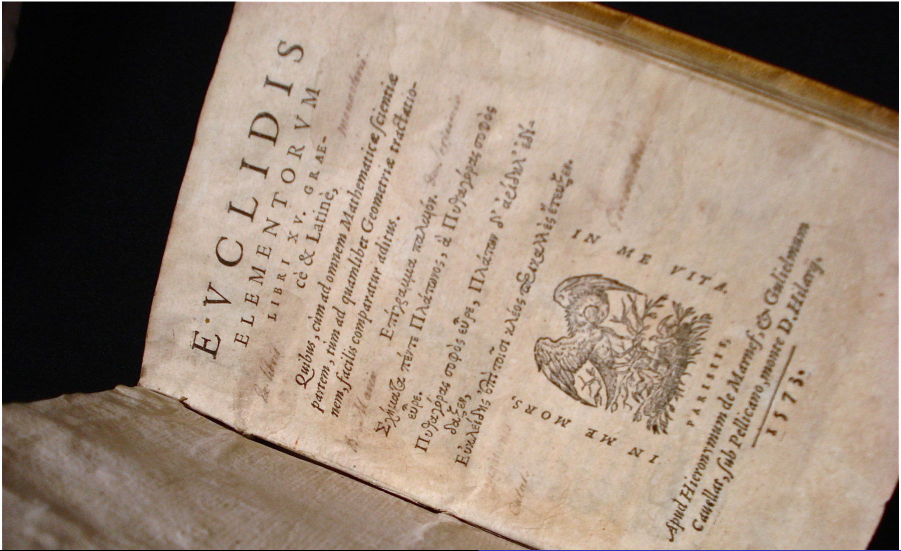




Complete (every statement in the language of the system can be proved or disproved)?

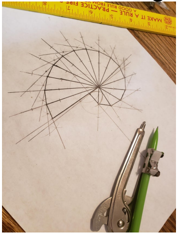
Axiomatic Systems: Euclid's Elements

CC BY-SA 3.0 *Euclid's Elements* 1573 Edition. Private collection Hector Zenil.



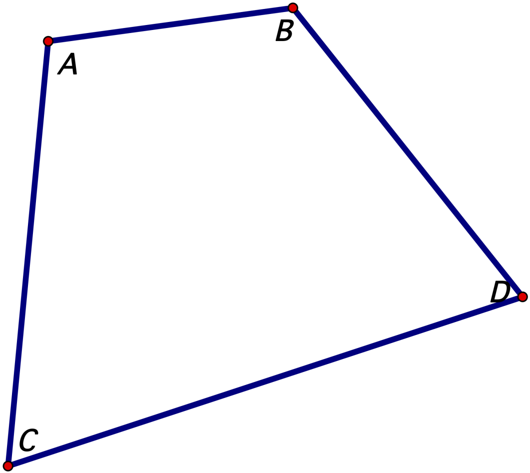
Geometric Constructions

- straightedge and compass
- Interactive Geometry Software (IGS) move geometric figure—configuration like the skeletal system of the human body or a mechanical device with interconnected parts, levers, and linkages—preserves dependency relationships to reveal invariants
- paper folding—isometries of the plane (linear transformations that preserve length)



https://www.reddit.com/r/GeometryIsNeat/comments/b4rgv4/a_ruler_and_compass_construction_of_a_pythagorean/

Measurements & Constructions: Quadrilateral Midpoints



fold midpoints
fold segments connecting adjacent midpoints

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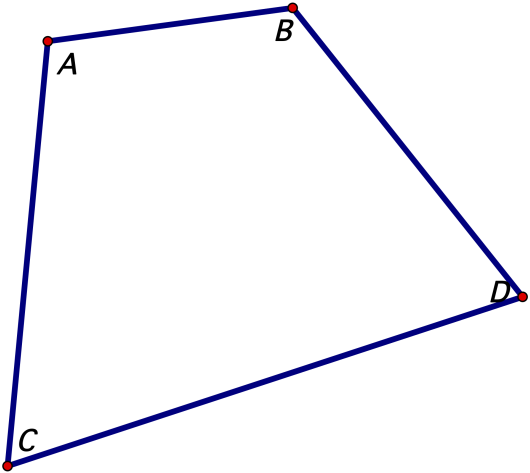

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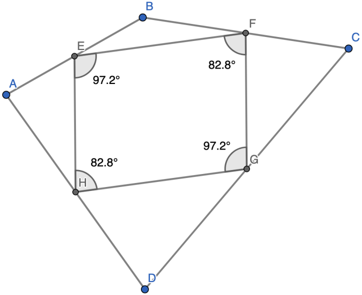
`https://www.geogebra.org/geometry`



Latvian/US mathematician Daina Taimina *Crocheting Adventures with Hyperbolic Planes*

Geometry is about how you view space. Take charge of it—it's yours. Understand how you see things and how you imagine things.

In front matter of *Experiencing Geometry: Euclidean and Non-Euclidean with History*, fourth edition by David W. Henderson and Daina Taimina