

# Measurements

How were circumference, area and volume formulas obtained via axiomatic and other perspectives and before coordinate geometry, metrics, and calculus II with analytic geometry?

- Eratosthenes and circumference of earth
- Archimedes and others' polygonal method for circumference and  $\pi$
- Archimedes' proof of area of circle by exhaustion
- Archimedes' proof of surface area of a sphere
- Archimedes and volume of a sphere (Cavalieri's principle)
- area of a triangle on the sphere and the sum of the angles



*"Pi what squared? Long John,  
you should be able to get this."*

<https://www.newyorker.com/online/blogs/cartoonists/pi-what-squared.jpg>



A set of small navigation icons typically found in Beamer presentations, including symbols for back, forward, search, and other slide controls.

## *Surface Area of Sphere*

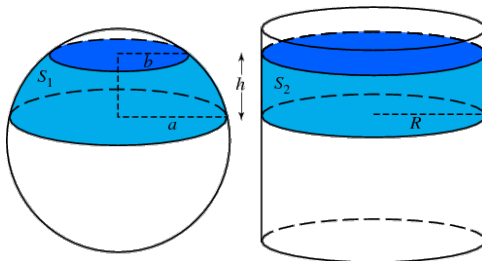
area of a sphere is 4 times the area of a great circle of the sphere  $= 4\pi r^2$

<https://www.youtube.com/watch?v=FB-acn7d0zU>

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<http://mathworld.wolfram.com/ArchimedesHat-BoxTheorem.html>

If a polygon is inscribed in a circle and revolved about an axis, forming a solid of revolution inside a sphere, then the surface area  $<$  surface area of sphere.

<http://staffhome.ecm.uwa.edu.au/~00003428/L7Archimedes1.html>

## *Volume of a Sphere Worksheet*

<http://www.cs.appstate.edu/~sjg/class/3610/ArchCav.html>

- Fill up the sphere with sand and pour it into the cylinder.  
Eyeball this: approximately what fraction of the cylinder does the sphere take up?

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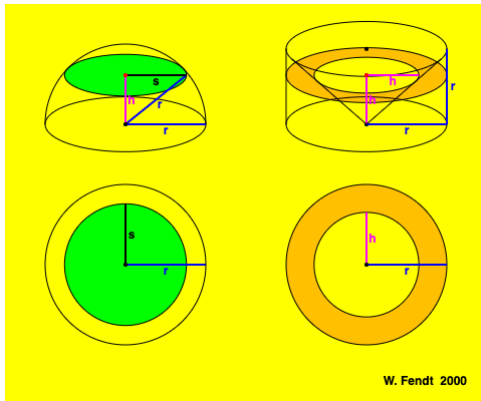
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cone volume + sphere volume:

$$\frac{1}{3}\pi r^2(h) + \frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2(2r) + \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3 + \frac{4}{3}\pi r^3$$

## Volume of a Sphere

Archimedes was trying to derive the formula for the volume of a sphere, so he could not assume this formula anywhere in his work.



<http://www.walter-fendt.de/m14e/volsphere.htm>

What familiar theorems are assumed?  $\pi r^2 - \pi h^2 = \pi(r^2 - h^2)$



Archimedes: Certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards.

[*The Method in The Works of Archimedes* translated by Heath]

<http://www.archimedespalimpsest.org/>

