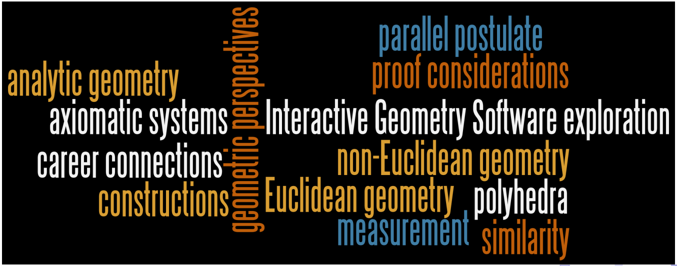
Project 2: Euclidean & Spherical—Course Topics

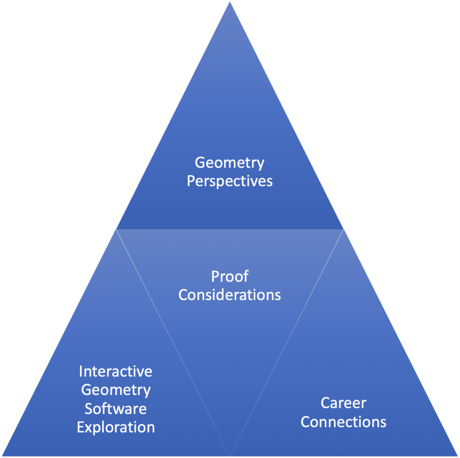
(Geometric Perspectives) I can compare and contrast multiple geometric perspectives...

- individual project
- review Euclidean content
- importance of topic
- diverse spherical perspectives
- references



Career Connections

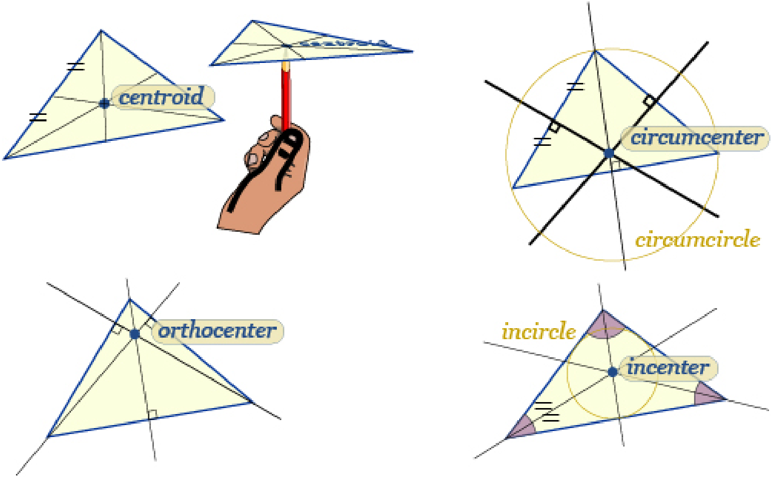
I can make connections between learning geometry in this class and teaching geometry in high school (for secondary education majors) or to geometry and focuses in my major(s) or intended career.



- CBMS statement “Active Learning in Post-Secondary Mathematics Education” (2016)
- MAA Instructional Practices Guide (2018)
- *Make It Stick: The Science of Successful Learning* by Peter Brown, Henry Roediger, and Mark McDaniel (2014)

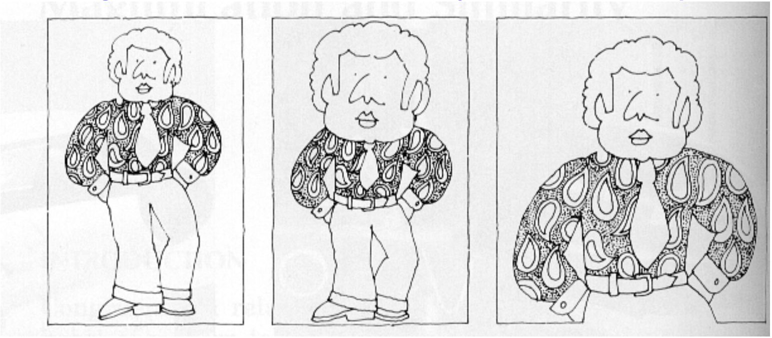
Fold the Angle Bisectors

Centroid, Circumcenter, Orthocenter & Incenter



https://www.mathsisfun.com/geometry/images/triangle-centers.svg

Congruence and Similarity: Same Shape?

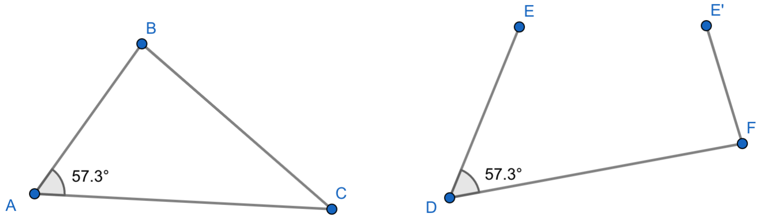


congruence
corresponding sides congruent and corresponding angles congruent

similarity
corresponding sides proportional and corresponding angles congruent

Congruence and Similarity: Euclidean SAS

- SAS: Two sides of the broken triangle *DEF* are the same length as the corresponding sides in $\triangle ABC$ ($\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$). The included angle is congruent ($\angle BAC \cong \angle EDF$). How many triangles can we create? Must they be congruent? If not, must they be similar? Why or why not?



https://www.geogebra.org/geometry/apkxfb5y

Congruence and Similarity: Proof of SAS

http://www.atheistrepublic.com/sites/default/files/styles/blog-featured-image/public/proof.jpg

Linear Transformations of the Plane

Rotation: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ Dilation: $\begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$ Horizontal Shear: $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

Projections: y=x line: $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ x-axis: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ y-axis: $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Reflections: y=x line: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ x-axis: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ y-axis: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Translation: $\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+h \\ y+k \\ 1 \end{bmatrix}$ Others: $\begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Isometries

isometry

mapping of a metric space onto itself that preserves distance between points

plane: translations, rotations, reflections, and glide reflections

$\rightarrow$ congruence

What preserves shape (but not congruence) from among dilation, shear, projection?

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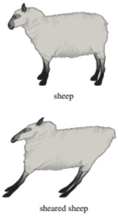
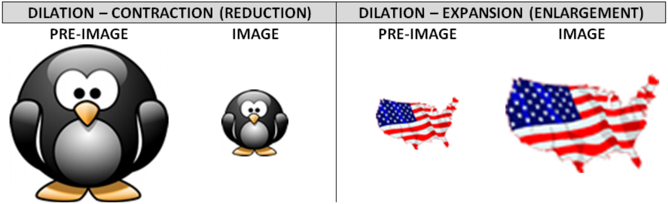




Linear Algebra and Its Applications by David C. Lay,

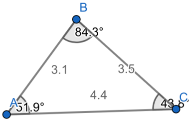
http://www.geometrycommoncore.com/content/unit2/gsrt1/teachernotes1.html

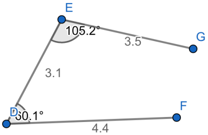
SAS Justifications in Other Contexts

- transformations
  `https://www.geogebra.org/m/bM5FkyFK`
- analytic/metric: Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$
- other axiom systems: SAS triangle congruence is a postulate not a proposition, or it is proven from one of the other congruence theorems

Congruence and Similarity: Euclidean SSS

- SSS: The three sides of the broken triangle are the same length as the corresponding sides in triangle ABC. How many triangles can we create? Must they be congruent? If not, must they be similar? Why or why not?



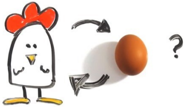


https://www.geogebra.org/geometry/bw4eagnh

Congruence and Similarity: Proof of SSS

SSS Justifications in Other Contexts

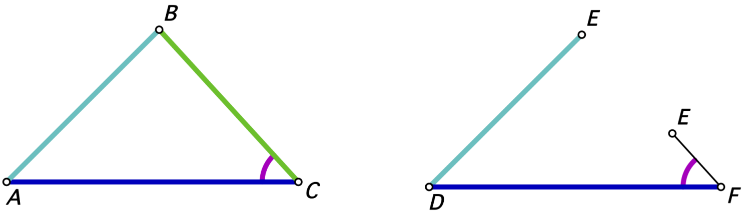
- transformations
  `https://www.geogebra.org/m/fskReBGs`
- analytic/metric: Law of Cosines: solve for C in $c^2 = a^2 + b^2 - 2ab\cos C$
- other axiom systems: SSS is a postulate not a proposition and is used to prove SAS
- Hadamard's argument:
  - Move triangle so that $\overline{BC}$ sits on $\overline{B'C'}$, both triangles sit on the same side, and assume for contradiction that A is not on A'
  - B is the same distance from A and A' so it lies on the perpendicular bisector of $\overline{AA'}$. C is similar, so $\overline{BC}$ is the perpendicular bisector.



www.brainpickings.org/2013/02/01/which-came-first-the-chicken-or-the-egg/

Congruence and Similarity: Euclidean SSA

- SSA: Two sides of the broken triangle *DEF* are the same length as the corresponding sides in $\triangle ABC$ ($\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$) . An opposite angle is congruent ($\angle ACB \cong \angle DFE$). How many triangles can we create? Must they be congruent? If not, must they be similar? Why or why not?



https://www.geogebra.org/m/E8cZQwjV

Congruence and Similarity: Proof of (not) SSA

Euclidean AA, SS, SA, ASA, AAS, HL

Work (no internet searches) to look for counterexamples, if they exist. You may use IGS. Fill with "yes" or a counterexample.

	congruence?	if not then similarity?
AA		
AAA		
SS		
SA		
ASA		
AAS		
HL		

A=angle, S=side, H=hypotenuse, L=leg of a right triangle