## Euclidean AA, SS, SA, ASA, AAS, HL

Work (no internet searches) to look for counterexamples, if they exist. You may use IGS. Fill with "yes" or a counterexample.

|  | congruence? | if not then similarity? |
| :---: | :--- | :--- |
| AA |  |  |
| AAA |  |  |
| SS |  |  |
| SA |  |  |
| ASA |  |  |
| AAS |  |  |
| HL |  |  |

$\mathrm{A}=$ angle, $\mathrm{S}=$ side, $\mathrm{H}=$ hypotenuse, $\mathrm{L}=$ leg of a right triangle

|  | congruence? | if not then similarity? |
| :---: | :--- | :--- |
| AA |  |  |
| AAA |  |  |
| SS |  |  |
| SA |  |  |
| SAS |  |  |
| SSS |  |  |
| SSA |  |  |
| ASA |  |  |
| AAS |  |  |
| HL |  |  |

## Propositions, Assumptions and Applications

Proof Considerations: I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.


## Applications of Congruence and Similarity

- every isometry in the plane is a product of at most three reflections (via SSS)
- two isometries which agree on three non-collinear points are the same on all points (via SSS)
- For triangles, SSS and more are lists of possible design choices in Euclidean geometry of the plane.
- More generally, in other geometries, or for more points, like for CAD or CAGD programming, when are design choices complete or when do we need more choices?


## Burden of Proof: Is the defendant a square?



## Euclidean Quadrilateral Design Choices

Investigate similarity in quadrilaterals (no internet searches). Look for counterexamples of quadrilaterals and write them on the board as you find them (e.g. SSSS with a counterexample) You may use IGS. Then consider what is the smallest amount of information you need to determine that quadrilaterals are similar?

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https://www.geogebra.org/m/zg3nzsRa
https://www.geogebra.org/m/Q8EYTUK2
https://www.geogebra.org/m/uKnpbCCc
SSAA and SSASA?

## SSAAA Similarity for a Euclidean Quadrilateral?

Assume two quadrilaterals $A B C D$ and $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ satisfy $\frac{\overline{A B}}{\overline{A^{\prime} B^{\prime}}} \cong \frac{\overline{B C}}{\overline{B^{\prime} C^{\prime}}}$ (i.e. SS similarity) and three pairs of the angles are congruent.

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- What, if anything can we say about $\angle C A D \cong \angle C^{\prime} A^{\prime} D^{\prime}$, and why?
- How about $\triangle A C D$ and $\triangle A^{\prime} C^{\prime} D^{\prime}$, and why?
- How about $A B C D$ and $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$, and why?


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What about SS congruence + AAA?

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What about SS congruence + AAA?
What about SAAAA and SAASA? SSSSA?

## Project 2: Euclidean \& Spherical-Course Topics

 (Geometric Perspectives) I can compare and contrast multiple geometric perspectives...- individual project
- review Euclidean content
- importance of topic
- diverse spherical perspectives
- references


