# Contents

# MAT 3610: Introduction to Geometry

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## 1.1 Course Goals and Objectives

Catalog: A study of the development of Euclidean geometry through multiple perspectives, including synthetic and metric. Topics to be considered include the nature of axiomatic systems and proofs, parallelism, similarity, measurement, constructions, polyhedra, utilizing appropriate technology, and at least one non-Euclidean geometry. The course will focus on concept development and connections among mathematical perspectives.



In order to foster concept development and connections among multiple perspectives, we will examine the foundations of geometry through the lenses of historical perspectives, intuition, reasoning and proofs, manipulatives, visualization, Interactive Geometry Software, and modern applications. To make additional explicit connections we will use a standards-based model based on associated course learning goals (standards):

• IGS Exploration

I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate that they seem to apply in a wide variety of examples.

• Proof Considerations

I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.

• Geometric Perspectives

I can compare and contrast multiple geometric perspectives, such as relationships among Euclidean and non-Euclidean geometries, axiomatic and analytic approaches, informal intuition and rigorous proof, or 2-D polygons and 3-D polyhedra, to name a few.

• Educational Connections

I can make connections between learning geometry in this class and learning and teaching geometry, especially in high school.

We will also see some connections to middle grades and college too. Future high school mathematics teachers often form the largest component of majors in this class and so this course is designed intentionally with these educational connections. As the MAA geometry study group recommends: "future high school mathematics teachers are best served by appropriately designed geometry courses for the general mathematics major that

also take into consideration the needs of secondary mathematics teachers, not by courses specifically designed only for future teachers" (CUPM, 2015). In line with these recommendations, this is a mathematics content course, which means that the primary goal is to stimulate intellectual growth and mathematical development of geometry at the 3000 level. Moreover, the mathematics covered in the course will be related in meaningful ways to materials that can be taken into future classrooms as well as various ways of teaching and learning geometry.

The content of the course (like axiomatic systems or the parallel postulate) is interwoven with the course learning goals (like proof considerations or educational connections) within activities that occur both inside and outside of class.

The first three learning goals are specifically credited within Reflection assignments while the last is credited within Project 4.

### 1.2 Required Resources

- Roads to Geometry Third Edition by Edward C. Wallace and Stephen F. West available for rental
- printouts of Project 4 to tape up
- scanning handwritten work: the course is designed so that you'll collate handwritten work from each classroom worksheet and project into a full size multipage PDF for submission in ASULearn. You can electronically annotate a PDF using an electronic stylus, or write on the paper copy I give you and scan and collate it into a PDF. If you have a phone or tablet, apps like Adobe Scan or CamScanner can work well. You can also use many printers or photo copiers to scan to PDFs—the school library lists that as an option and they can help:

https://library.appstate.edu/services-search/print-zone-tech-help. PDFsam on a PC or Preview on a Mac can collate multiple PDFs together, like for a project.

• reliable access to technology, software, and high speed connectivity: The work you're going to do outside of class you'll be accessing though your internet connection with a device, so it will be really important to have access to a dependable high-speed internet connection and a good computer that can run everything we'll need. The software is free, including GeoGebra Geometry and Microsoft Excel—any student with a valid Appstate email address can access Excel on their own computer or on campus computers. For optional office hours, we'll use Zoom videoconference software with breakout rooms. Flexible browsers that will play common media formats from various sources such as from webpages, Google Drive, YouTube, and ASULearn, including interactive videos, a taxicab treasure hunt, and html5 apps are also something we'll use. You may need some flexibility in browsers so that if one browser is incompatible, you can try another. ASULearn components work best from scrolling through the activities themselves on a computer—the Moodle mobile app does not always show everything as designed, both for visibility and for due dates. Hybrid course students are expected to have or acquire proficient computer skills and to resolve their own technology problems related to computers or internet access to turn things in before deadlines. Work early and leave yourself enough time before any strict deadlines to allow for technical issues that may arise. For example, ASULearn will typically be undergoing monthly system maintenance on the last Thursday of each month. https://asulearn.appstate.edu/?redirect=0 announces downtime and http://Support.appstate.edu and https://confluence.appstate.edu/display/ATKB/ Appalachian+Technology+Knowledge+Base can help.

## 1.3 Assignment Types, Grades, and Policies



Understand Misconceptions & Apply review my feedback projects Solidify & Make Connections reflections final assessment

#### • Effective ASULearn Engagement 30%

ASULearn completion activities may be ones where you can manually mark the activity as completed or are earned for a good faith effort when you access an activity or receive a proficient grade by a deadline. Attempt items for completion and take interactive video notes by the listed date when possible as the material builds on itself. Generally, the percentage of completed activities determines the overall engagement grade. While many items have strict deadlines, there is flexibility built in and multiple pathways for success interactive videos have multiple chances to succeed, many ASULearn items can be completed ahead, plus, to accommodate for emergencies, the lowest 3 completion assignments are dropped. In addition, evidence of deep and conscientious engagement in the activities is a part of the grade, which may be adjusted at the end of the semester to reflect it. ASULearn provides detailed reports. Activities may include web pages, PDFs, files, interactive videos, surveys, forums, turn-ins, begin projects and reflections, select a topic, and peer review. Note that the segment headings in the ASULearn engagement activities is where we begin those course topics through videos and more and we'll continue them throughout the semester in many cases.

#### • Projects 30%

There are 6 projects over the course of the semester, which include problem sets as well as research projects that you will also share with your classmates. They are designed to apply course learning goals to the content of the course in novel settings and from numerous points of view in order to practice the material and make connections. The Educational Connections learning goal is specifically credited within Project 4. Your work must be turned in to the ASULearn assignment on or before the due date and time because we will often talk about them during class. Problem Sets earn a numerical grade toward your course average. To accommodate issues that may arise, you can revise one project as you reflect on it to replace its grade. Some days are lighter than others and it will help you to progress on upcoming activities in advance, especially major assignments.

#### • Reflections 20%

Talking time to look back at what you have done and make new connections allows you to solidify knowledge and put your understanding in context. Reflections are expositions, typically the equivalent of 2–3 pages long, single-spaced typed text, and can also include images and references. Reflections connect content in the course to course learning goals. They are quite flexible so that you can follow up on and make connections to your own interests. For each reflection, list the learning goal you want me to assess from among IGS Exploration, Proof Considerations, and Geometric Perspectives, reflect on every single component of it every word in the learning goal—and give at least one example we covered in or between classes that you feel best showcases the goal and your understanding of it. In addition, reflect and personalize. For example, you might focus on your own development as related to the goal, what you are still working on, and/or you could conduct research to find additional new connections. Reflections are individual expositions because of the personalization. For the first reflection, you will receive feedback from peers and myself and have a chance to revise it and there are other revision opportunities on reflections too. Other than those, there are no late reflections accepted. Success in reflections is tied to rubric standards, rather than a percentage correct or point system. This will allow you (and I) to identify strengths and weaknesses early in the class. Your work must be turned in to the ASULearn assignment on or before the due date and time, but there are multiple opportunities to work on problem areas and succeed. In reflections, we'll employ Star Wars terminology as a metaphor—Padawans are training to one day become a Jedi. Jedi complete their training and pass the Jedi Trials to become full members. The numerical version of this part of your grade is calculated as the percentage of learning goals achieved.

#### • Final Assessment 15%

Designed to help you solidify and make connections, we'll have an assessment during our assigned time at finals. There will be an interactive video notes portfolio component, an individual component, and a component where you can work in groups. You work alone until I collect the individual part of the assessment and say it is "group time." Then you may continue to work alone or in groups (or a combination!). The idea is to give you opportunities to communicate course content with your peers, since this is one of ASU's main educational goals. As listed at the registrar "An instructor may only change the examination schedule for a course with the approval of the department chairperson and the dean."

#### • Effective Class Engagement 5%

When I was designing this hybrid class, ASU's Center for Excellence in Teaching and Learning for Student Success designated that "Face-to face component is not a lecture but provides time for discussion, demonstrations, problem-solving, and higher-level thinking and collaborative activities. Class time is used to apply course content in ways that can only be accomplished when everyone is together in the same place." https://cetlss.appstate.edu/teaching-learning/course-delivery-options.

Effective class engagement is essential to our course integrity and attendance is required at ALL such classes, with the exception of legitimate or excused absences. If you expect to miss more than 10% of classes due to university sponsored activities or other reasons then I advise you to drop the course. Any student who wants to obtain an "excused absence" for less than 10% of classes must meet certain responsibilities, including contacting me on the ASULearn forum in advance when possible, providing official documentation, and making up any possible work in advance.

If the university cancels classes or changes them, continue progressing on the work due on ASULearn and check ASULearn for any updated info, which may include plans for the missed class such as Zoom meetings or individual activities.

It will be useful if you bring a computer, tablet, or phone to classes that can access webpages—if not, you can still participate in other ways.

ASU prepares students to employ various modes of communication that can help communities reach consensus or respectful disagreement: successful communicators interact effectively with people of both similar and different experiences and values and in this class you will practice oral and written communication during class by interacting with your peers and me. Regardless of gender, political party, race, religion, sexuality, or more this class is to be a welcoming environment, and so I want you to be sensitive and respectful to each other in upcoming discussions. Keep it a safe place to express meaningful ideas and opinions. Actively listen to others and encourage everyone to participate. Part of the welcoming environment is to keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter. Performing activities that detract from the welcoming environment or distract your neighbors or me will result in a lowered grade. Asking or answering related and thought-provoking questions, coming up with creative ways of thinking about the material, and explaining the material to others are some examples of positive class engagement that will increase your grade.

\* Accommodations in the determination of your final grade may be made for extenuating circumstances that are officially documented to prevent you from completing work early or on time.

The grading scale is:  $A \ge 93$ ;  $90 \le A - < 93$ ;  $87 \le B + < 90$ ...

## 1.4 Tentative Calendar

This class has two of our three credit hours as in-person meetings. You'll have daily work between our classes but you have the flexibility to work ahead to meet the deadlines if that is better for your schedule: plan to spend 3.5–5 hours between classes, on average, as per the University-wide Statement on Student Engagement with Courses. Our hybrid class is officially designed by the registrar and scheduled by the university for our third hour to be a part of the activities between classes.

Details for assignments and in-class activities and any changes are on

https://www.appstate.edu/~greenwaldsj/class/3610/s24.html

and ASULearn. While activities on specific course content are listed below, topics are not limited to that time period. For instance, constructions are introduced early on but continue throughout the semester.

	<u></u>			<b>D A</b>
	Class Monday	Between Classes	Class Wednes-	Between Classes
		(by 1:55pm Wednesday)	day	(by 1:55pm Monday)
1/17			active learning	-axiomatic systems and construc-
			worksheet	tions 1 interactive video
				-3610 intro interactive video
				-turn in worksheet
				-obtain rental book
				-read the syllabus
				-add ASULearn profile pic
				-Zoom update & profile pic
1/22-	learning goals	-axiomatic systems and construc-	axiomatic sys-	-Project 1
1/24	worksheet	tions 2 interactive video	tems and con-	-turn in worksheet
/		-IGS intro interactive video	structions 1	
		-begin Project 1	worksheet	
		-turn in worksheet		
		-get to know posting		
1/29-	axiomatic sys-	-congruence and similarity 1 inter-	congruence and	-congruence and similarity 2 inter-
1/31	tems and con-	active video	similarity 1	active video
	structions 2	-select topic for Project 2 and be-	worksheet	-turn in worksheet
	worksheet	gin working on it		-review and reflect on axiomatic
		-turn in worksheet		systems and constructions
2/5-	congruence and	-Project 2	Project 2 eleva-	-Euclidean and spherical perspec-
$\frac{2}{3}$	similarity 2	-turn in worksheet	tor pitch on Eu-	tives interactive video
	worksheet		clidean items	-begin Reflection 1
2/12-	spherical per-	-Beflection 1	spherical angle	-peer review Beflection 1
$\frac{2}{14}$	spectives work-	-begin Project 3	sum and AAA	-turn in worksheet
	sheet	-turn in worksheet	worksheet	
2/10_	Pythagorean	-Project 3	Pythagorean	-Pythagorean theorem interactive
$\frac{2}{10}$	theorem 1	-turn in worksheet	theorem 2	video
	worksheet		worksheet	-select topic for Project 4
	worksheet		worksheet	turn in workshoot
				review and reflect on congruonee
				and similarity
2/26_	research guide	analytic geometry and matric per	analytic goomo	propage for Project 4 procents
$\begin{vmatrix} 2/20 \\ 0/99 \end{vmatrix}$	for Project 4	spectives interactive video	try and motric	tions and bring printout to tape up
2/20		road Reflection 1 feedback	porspectives 1	turn in workshoet
		-reau memerion r recuback	workshoet	
2/4	Drainat 4	partian (if monded) and the '	Draiget 4	turn in Decient 4 1
$\begin{vmatrix} 3/4 - \\ 3/6 \end{vmatrix}$	roject 4 presen-	-revise (ii needed) and turn in	Froject 4 presen-	-turn in Project 4 peer review and
3/0	tations part 1	Project 4	tations part 2	sen-evaluation

3/18-	analytic geome-	-Reflection 1 revision (if needed)	polyhedra work-	-polyhedra and angle defect inter-	
3/20	try and metric	-begin Project 5	sheet	active video	
	perspectives 2	-turn in worksheet		-turn in worksheet	
3/25-	measurement	-measurements and angle sum in-	earth and uni-	-Project 5	
3/27	worksheet	teractive video	verse measure-	-turn in worksheet	
		-turn in worksheet	ments worksheet		
4/1-	equidistant wa-	-turn in worksheet	proof worksheet	-Reflection 2	
4/3	ter reservoir	-review and reflect on polyhedra,		-turn in worksheet	
	worksheet	analytic/metric perspectives and			
		the Pythagorean theorem			
4/8-	hyperbolic 1	-parallels 1 interactive video	hyperbolic 2	-parallels 2 interactive video	
4/10	worksheet	-begin Project 6	worksheet	-Reflection 2 revision (if needed)	
		-turn in worksheet		-turn in worksheet	
4/15-	hyperbolic 3	-parallels 3 interactive video	hyperbolic 4	-Project 6	
4/17	worksheet	-turn in worksheet	worksheet	-turn in worksheet	
		-review and reflect on measure-			
		ments and parallelism			
4/22-	Desargues' theo-	-projective geometry interactive	reflections on	-survey and evaluations	
4/24	rem worksheet	video	geometry work-	-turn in worksheet	
		-Reflection 3	sheet	-begin final assessment guide	
		-turn in worksheet			
4/29-	review activities	-Reflection 3 revision (if needed)	concluding	-prepare for final assessment	
5/1			activities	-prepare to turn in video notes	
5/3	timed assessment during assigned time at finals—video notes due + individual and group components				
2-	(optional) revise and reflect on one project to replace its grade				
4:30	(optional, if needed) revise and reflect on one reflection to replace its completion status				



## 1.5 Course Communication, Office Hours, Where to Get Help, and Additional Policies

• I encourage you to talk to me and your classmates often in and out of class. Outside of class, I can help in office hours on Zoom, internet allowing, Sunday, Tuesday, and Thursday 7-7:45pm and Monday and Wednesday 12:30-1:30pm via the link in the Need Help Forum on ASULearn from https://asulearn.appstate.edu/course/view.php?id=167038

You can come in to Zoom to work on future assignments, working along and asking questions as they arise, or you can ask me questions about items you already turned in, ask me questions related to course content or structure, or ask me to go over a concept. Keep me informed about any problems and I also want to hear when things are going well! We can both write on a whiteboard and share our screens. You do not need to make an appointment to meet with me in Zoom, just drop by and I am happy to help! During class and in online communication, use standards of professionalism and collegial communication. This course is

to be an environment in which everyone feels comfortable asking questions, making mistakes, offering good guesses and ideas, and is respectful to one another.

Please use a salutation of Dr. Sarah, my preferred name, in communications with me. If you can't make it to Zoom, you can contact me with a professional message on the Need Help Forum on ASULearn, which I'll try to answer once a day, including the weekends. If you haven't heard back from me yet, only message again if you are addressing a new and separate issue or retracting a previous message. Private conversations with me should occur during office hours or via the forum, not during class time. In Zoom hours, I can set up individual breakout rooms for private conversation with me. Except for extreme emergencies, all private written communication must be handled through your private forum, with just you and I, on ASULearn rather than e-mail. I prefer that you use the Zoom hours as it is easier to discuss material in person.

• The Conference Board of the Mathematical Sciences (CBMS) published a statement titled "Active Learning in Post-Secondary Mathematics Education" about the importance of "classroom practices that engage students in activities, such as reading, writing, discussion, or problem solving, that promote higher-order thinking" and our classroom is modeled after that. The purpose of engagement is to learn and practice course content and learning goals, and develop critical thinking and problem-solving skills. Making mistakes is integral to the learning process—the key is to try to continue to engage rather than give up—and this course is to be an environment in which you ask questions and offer good guesses. It is on purpose that there are problems that don't look exactly like what we did previously in order to provide you with rich settings to explore in order to learn deeply.

I do not expect you to be able to solve all the issues immediately. Instead, I want to see what you can do on your own. Out in the real world, this is important, since no matter what job you have, you will be expected to seek out information and answers to new topics you have not seen before. This may feel uncomfortable and frustrating. I understand this and want to help you through the process. It helps to remember that there are no mathematical dead-ends! Each time we get stuck, it teaches us something about the problem we are working on, and leads us to a deeper understanding of the mathematics. In the real world though, you are not expected to face your work alone, and in this class is similar.

Asking questions, and explaining things to others, in or out of class, is one of the best ways to improve your understanding of the material. I am always happy to help and will try to give you hints and direction to help you understand the material. At times though, to encourage the exploration process, I may direct you to rethink a problem and to come back to discuss it with me again afterwards. This occurs when I believe that the struggle to understand is imperative for your deep understanding of the material.

- Academic integrity is a fundamental part of the course, which includes meeting deadlines, regular communication, and giving proper reference where it is due. These are essential to course integrity. Feel free to talk to me or each other if you are stuck, but when writing up work, be sure to give acknowledgment where it is due. Submitting someone else's work as your own (PLAGIARISM) is a serious violation of the University's Academic Integrity Code, which defines: "Plagiarism includes, but is not limited to, borrowing, downloading, cutting and pasting, and paraphrasing without acknowledgement, including from online sources, or allowing an individual's academic work to be submitted as another's work."
- I believe that you have the capability to succeed in this course. If you find that you are spending fewer hours outside of class than the university recommends, you can probably improve your understanding and grade by studying more. If you are (on average) spending more hours than these guidelines suggest, you may be working inefficiently; in that case, you should come see me.
- Prerequisites: I will assume you have facility with the following that you developed in the prerequisite of MAT 2110 Techniques of Proof: methods of proof used in mathematics, writing mathematics, searching for mathematical content and sources, and careers in mathematics, as well as from MAT 2110's prerequisite MAT 1120 Calculus With Analytic Geometry II: analytic geometry as well as techniques of integration and their connection to arc length, area, and volume.
- During class, use of technology is allowed only when it is related to our class. Photos or video or audio recordings may not be taken in class without prior permission. Sharing our course materials, ASULearn

items, photos, video or audio recordings may not occur without prior permission. There is no eating or drinking allowed in the classroom but you may step out if you need to hydrate or similar. Many activities are designed to be completed during class and you are responsible for all material and announcements, whether you are present or not. You are also responsible for announcements made on the web pages, so check them often.

- We adhere to University-wide syllabus and policy statements: https://academicaffairs.appstate.edu/resources/syllabi-policy-and-statement-information and University policies like on https://policy.appstate.edu/Policy\_Manual
- I want you to be informed about your choices regarding what you tell me about certain types of sensitive information. In situations where students disclose experiencing an act of interpersonal violence to their instructor, faculty are required to report that to the campus Title IX Coordinator, who then reaches out to the student by email offering support services. I care about you and want you to get the resources you need. I'm happy to talk with you if you decide you want that, but please be aware that if instead you'd like to explore options with someone who can keep your information totally confidential, I highly recommend the Counseling Center at 828-262-3180. They offer walk-in hours as well as after-hours coverage: http://counseling.appstate.edu.
- Appalachian Cares is a place to find updates about matters of student health and safety http://appcares.appstate.edu/.
- Belk Library offers a wide array of research services including access to their digital media studio (with assistance available), video and audio recording rooms, Research Advisory Program sessions in which you meet individually to work with a librarian, Library guides, study spaces, and online workshops, like http://library.appstate.edu/gethelp/rap
- The writing center can help you with drafts of your reflections https://writingcenter.appstate.edu/
- Disco Student Learning Center offers academic resources that will complement and enhance classroom experiences by helping students become acquainted with their studies and learn how to learn effectively. For example, you can meet with a study skills specialist to discuss your goals and develop strategies and study plans to meet those goals or arrange for tutoring. "D.D.," or Dauphin Disco, was one of the three people who, in 1899, founded Watauga Academy known as App State today. The word disco is Latin for learn.

#### 1.6 Advice from Prior Students

- The advice I would give future students taking this class would be to take good notes on the interactive videos and go to office hours if you have questions on anything. Dr. Sarah is always willing to help and answer questions.
- Pay attention during the interactive videos. Learn the propositions, common notions, postulates, and definitions. Also, make sure you understand the proofs.
- I would tell students to ask questions often and to dig into the material beyond what they would typically do, like proofs in videos and Dr. Sarah's personalized worksheet feedback.
- Just stay on top of classwork. Make sure to actively listen to the lecture videos and show up for class. The worksheets done in class really help to reinforce ideas learned in the videos and explain concepts that didn't click while watching the lecture videos.
- Do the interactive worksheets they're super helpful!
- Interactive video notes help when reviewing for projects or reflections.

- Keep up with upcoming assignments and take the projects seriously, as they really solidify the information being taught.
- Start on the projects ahead of time and ask questions. Dr. Sarah is always super helpful and understanding.
- I loved the way the class was set up with interactive worksheets and notes and then projects to solidify and reflections to go back over. It really helped me to solidify the information.
- Make sure you do something with all of the words in the reflection descriptions for each learning objectives. Also make sure you do your work on time and start projects before they're due.
- For the reflections, they are not nearly as scary as you would think, but I would recommend using office hours to discuss your papers with Dr. Sarah. She gives amazing feedback!
- Go to office hours to get help!
- Study Euclid's elements closely. I also recommend not holding on to the ideals of "traditional teaching" as this class allows students to break free from that and truly explore mathematics.
- Definitely work in groups and have a community in the classroom. The majority of us were education majors but this is one of the first classes I've had that actually made me feel like I was a part of something bigger.
- Go over the final assessment guide and ask questions.
- Don't stay too frustrated if you don't understand some aspects of the course at first or why you are learning the way you are. It made sense to me half-way through the course.
- I would suggest that students enter the class with an open mind. I was a little nervous about taking this class because geometry wasn't something I was super interested in, but this class has definitely changed my opinions on this.
- Be ready to challenge your intellect. Do not be afraid of exploring and being creative.
- If I could advice to future students, I would encourage them to use office hours and class meetings as much as possible. Dr. Sarah was always there when I needed to ask any questions or obtain help. You should be willing to think outside of the box, for your whole outlook on geometry will certainly change. It is an amazing course, and I am glad I had the chance to take it.

## 1.7 Acknowledgments

CUPM, 2015. Committee on the Undergraduate Program in Mathematics (CUPM) geometry study group report. William Barker, Frank Farris, Sarah Greenwald and Gerard Venema, Mathematical Association of America.

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## 1.8 Instructor Bio

I am a full Professor of Mathematics, and I am also an affiliate of Gender, Women's and Sexuality Studies (GWS) and the Math and Science Education Center (MSEC), investigating the connections between mathematics and society. My PhD in Riemannian geometry is from the University of Pennsylvania. I am married to the bassist Joel Landsberg. In our spare time, we like to travel, hike and conduct genealogy research. In addition to my own personal genealogy, I like to give back to the broader community, and in this context, I am also affiliated with ASU's center for Judaic, Holocaust and Peace Studies. Some of what I like about mathematics is also what I enjoy about genealogy—the sense of exploration, discovery and aha moments that come with lots of patience and effort.