MAT 3610 Video Interactions

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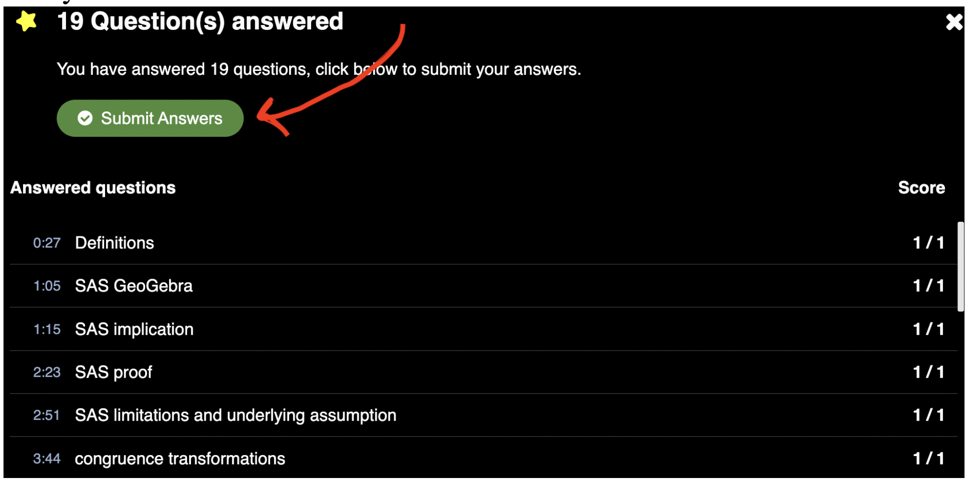
These are interactions in some of the interactive videos I created. In the interactive video, the video pauses, asks a question, and requires a response to proceed. To earn credit we watch the entire video and submit the correct answers via the green "Submit Answers" button at the very end of the video, the one that shows all the questions we have answered—we use the check feature on interactive questions in order to help and can redo the responses until they are correct.



Each video includes directions "to take notes on that you can bring with you to class: pause regularly to take notes that you can bring with you to class especially on concepts, proofs, Interactive Geometry Software and other visualizations, and any remaining questions." Above each video we include resources needed such as GeoGebra exploration links and Euclid's *Elements* Book 1. The added symbol of ✎ are items specifically directed for notes.

axiomatic systems and constructions 1 interactive video

- ✎In your notes, which you'll turn in at the end of the semester, write the title of this video "Axiomatic Systems and Constructions 1" as well as instructions for assembling a peanut butter and jelly sandwich that a robot could understand and follow. Assume that you have the needed materials already.
- How do we get completion credit for videos?
- Individual questions are repeatable and videos continue to be available later and are repeatable too. Which are true about your own notes on videos?
- If you've never played Minesweeper before, try a beginner game in the address I've listed just above the video but still inside this activity or from elsewhere.
- ✎ Write out a proof that B1 is a number in your notes. Be sure to refer to the Axioms. Also make use of C1=1 and C2=mine.
- Make sure that your proof has both axioms in it and consider improvements that would help the robot follow the proof and be convinced by it.
- ✎ Write the axioms of incidence geometry in your notes.
- Consider how to prove the statement that there exist at least three distinct lines not all through any common intersection point.

- This proves that we have at least three distinct lines, so now consider how to prove that they don't share a common intersection point.
- ✎ Write out a proof in your notes for incidence geometry that there exist at least three distinct lines not all through any common intersection point.
- ✎ Sketch an incidence geometry in your notes with exactly 3 points that satisfies all the axioms.
- Consider how this Fano model with 7 points also satisfies the axioms of incidence geometry.
- Why isn't incidence geometry complete?
- Take a look at the definitions in Euclid's Elements, especially definitions 1, 2 and 4, either on the first page of the handout, if you have it, or online otherwise, at the link I put above this video.
- Next, take a look at Euclid's First 3 Postulates, the axioms, either on the back of the first page of the handout, if you have it, or at the link otherwise.
- ✎ In your notes, roughly sketch visualizations of Euclid's Postulates 1, 2, and 3.
- Using the quadrilateral I handed to you, or by making one yourself, fold so that A and B overlap and the lines emanating from them do to. Then pinch fold at the midpoint.
- Pinch fold the other 3 midpoints.
- Fold the segments connecting adjacent midpoints. What do the 4 segments connecting adjacent midpoints form?
- Open up GeoGebra Geometry. You can use one of the links from above the video.
- Use the segment tool to create a quadrilateral and get practice working in the IGS (Interactive Geometry Software).
- Under Construct, use the "Midpoint or Center" to construct the four midpoints.
- Use the segment tool to connect adjacent midpoints.
- Under Measure, use Angle and select 3 points at a time to measure each angle in what seems to be the parallelogram formed by the midpoints. So you'll have 4 angles total.
- Move one of the blue points to see the dynamic nature of the IGS and see that we have a wide variety of examples where we still seem to have a parallelogram.
- ✎ In your notes, write some benefits and challenges of IGS (Interactive Geometry Software) and roughly sketch the construction in your notes.

3610 course intro interactive video

- What should you call me in class and in all communications?
- Is a growth mindset an important part of class?
- Open up course calendar above the video but still inside this activity.

- Even through there are hard deadlines, where is flexibility built in to the course?
- Consider the deadlines and activities and assignments on the PDF calendar.
- Consider the active learning environment and whether you can bring a laptop, tablet, or phone with you to classes
- Consider the welcoming environment during class.
- Our hybrid class is officially designed by the registrar and scheduled by the university for our third hour's time in and out of class to be a part of the activities between classes. The university recommends 2-3 hours outside of class for each credit hour (c.h.) and we add the 3rd hour's time in too: that gives $2 \cdot 3$c.h. $+ 1$ to $3 \cdot 3$c.h. $+ 1$ as weekly engagement time outside of our Walker Hall classes. Since there are two of those, divide these ranges total by 2 to see the university recommended hours between each class.
- Where do we find the completion items and how do we know when they are completed?
- Are video notes important?
- How do we earn completion for interactive videos?
- What are some of the features of the interactive videos?
- Even with completion, you might have some aspects that need improvement on a worksheet. How can you tell what is incorrect?
- Consider the 6 projects and their due dates in the PDF calendar from above the video.
- Consider that the content of the course is interwoven with the course learning goals within activities that occur both inside and outside of class.
- How do you achieve completion in a reflection?
- While worksheets, begins, projects, reflections and the final exam have strict deadlines, there is still flexibility built in and multiple pathways for success. You can recover a missed project with an optional revise and reflect assignment, there are optional reflection revisions, and the lowest 3 completion items are dropped. Other activities are designed to have you complete them in the order they are listed but also have second chances. We'll all be together in Boone for Project 4. Consider the grading, deadlines, and the PDF calendar.
- Consider these and whether there are any you'll review (tutoring or online sources are great for review)
- Where can you go for help outside of class?
- What is some common advice from prior students?
- Consider the graphics

axiomatic systems and constructions 2 interactive video

- Consider what we will assume is given and how we will use contradiction to prove that if l and m are two distinct lines that meet, then they meet at a unique point
- Consider the structure of the proof here and how we have shown A implies B by contradicting A and $\sim B$.

- ✎ Compare the images of the postulates (Euclid's names for the axioms) with what is in Euclid's book 1 for the postulates. Then copy the statements of postulate 1, postulate 2, and postulate 3 in your notes to help solidify them, since we'll be using them today.
- Fold an equilateral triangle yourself. First fold a long line of symmetry, then take one of the top corners and pinch fold it as you bring the other top corner down to the line of symmetry. For the third fold, fold the short edge that is left up to the fold we just made and match edge to edge. Fold it. Then tuck the remainder away.
- Download or open the GeoGebra Geometry App at `https://www.geogebra.org/download` or `GeoGebraGeometryathttps://www.geogebra.org/geometry`
- ✎ Test out the equilateral triangle construction in GeoGebra. Start with the segment tool to create the given segment. Then use the circle tool to create the 2 circles. Then the segment tool again to connect to one of the intersection points and create the triangle. Finally the move tool to drag A. Also sketch a picture in your video notes.
- In the construction we create a circle of radius AB and center A. What postulate lets us construct this circle?
- ✎ In your notes, write down a possible fix to the implicit assumption that C exists—we will see later in the semester that this will be a problem for us in other geometries. Write "if the sum of the radii of two circles is larger than the line joining their centers, then they intersect."
- What does postulate 1 say on the handout?
- What property of circles will be useful in the proof?
- Read definition 15 to see this.
- ✎ Write out the proof of I-1 in your notes to help you internalize it.
- Consider the nature of this proof and whether a reader could sketch the picture and understand the justifications from only the 2 items: the proof shown here plus the Euclid's book 1 handout.
- ✎ Fold a perpendicular yourself and sketch the fold in your notes.
- ✎ Write out the proof so far and a sketch in your notes too. Compare with the reasonings in Book 1 to help solidify.
- ✎ Here and in your notes, what is the difference between Postulate 1 and Proposition 1?
- Write out the rest of the proof of I-11 in your notes and sketch the picture with the tick marks.
- What is logically equivalent to the statement of I-11?
- Where does G first arise from in this construction of I-11?
- Consider Federico Ardila's axioms
- Consider Wile's predicament and if you can help him.

IGS intro interactive video

- ✎ Write down the themes of geometry worlds, dynamic geometry movement, and the power of community in your notes.
- Try it in GeoGebra.
- Respond here and write in your notes how we access the Intersect feature
- Continue the angle bisector construction
- Measure the angles
- Do the circles ever disappear?
- ✎ Roughly sketch the construction as well as the two triangles in your notes and consider how I-8 could help here.

congruence and similarity 1 interactive video

- ✎ Write the definitions of congruence and similarity in your notes. Be sure that you take notes for videos in a way that you can bring with you to future classes.
- Open the SAS GeoGebra above the video and leave E alone but drag E' to complete the triangle.
- ✎ Write in your notes and respond: What does SAS in a Euclidean triangle lead to?
- ✎ Read both CN4 and the statement of I-4 [SAS] in Euclid's Book 1, and write down what CN4 says in your notes.
- ✎ Write out the proof that SAS leads to congruency on something you can bring to classes as you consider limitations and underlying assumptions.
- Which is true regarding a limitation of this proof relating to the underlying assumption of BC and EF?
- ✎ In your notes, identify the underlying assumptions of being able to superimpose as well as having uniqueness of line segments.
- Consider what transformations of the plane preserve congruence of figures, i.e. are isometries
- ✎ Sketch a rough picture of the sheared sheep and label it as a shear
- Consider what transformations of the plane preserve similarity but not congruence
- Open and explore the SAS transformations GeoGebra by Tim Brzezinski above the video
- ✎ Write down the law of cosines and consider that if we have a, b and the included angle C then it will give us a unique positive square root value for c
- Read over Postulates 1 and 4 and Propositions 4, 10 and 11 in Euclid's Elements.
- ✎ Write a paragraph proof in your notes, with all the details, including an introduction, sentence connectors, and a conclusion [i.e. turn the sketch that any point on a perpendicular bisector is equidistant from the endpoints into a proof]

- Compare your proof and sketch with the tickmarks and angle showing SAS assumptions. The proof is also in the video slides.
- ✎ Sketch a rough picture of the intersection of the perpendicular bisectors of AB and AC for triangle ABC in your notes and consider what you can say about it
- ✎ In your notes, write out reasoning that the 3 perpendicular bisectors of a triangle intersect at one point, the circumcenter, the center of a circle going through the 3 vertices, but do not write a complete proof, plus sketch a picture.
- Consider AA similarity and SAS proportionality.
- ✎ Sketch a picture and label all the givens including the 2 sets of proportional sides and the included angle that is congruent.
- Review the application of SSS in I-11.
- ✎ Look for 2 places to complete the triangle in the SSS GeoGebra above the video and sketch them in your notes.
- Read Proposition 7 in *Euclid's Elements*.
- ✎ Open and explore the SSS transformations GeoGebra by Tim Brzezinski above the video. In your notes, write the comment about reflections.
- If $a = 3$, $b = 4$, and $c = 2$, use the law of cosines to solve for C, the angle opposite c
- Consider Jacques Hadamard's argument
- ✎ Open and explore the SSA GeoGebra by t. picone above the video and sketch and label the results in your notes
- Which of the following lead to congruence?

congruence and similarity 2 interactive video

- How is SSS congruence applied in the proof that an isometry is the product of at most 3 reflections?
- Consider what is the smallest amount of information we need to determine that quadrilaterals are congruent or similar?
- ✎ Take a look at SSSS quadrilaterals above the video. Explore the GeoGebra by sliding and then by dragging the one vertex. In your notes, sketch pictures and annotate what is the implication of the exploration. Does SSSS imply congruence or similarity in quadrilaterals?
- ✎ Look at AA triangles above the video. Explore the GeoGebra by the slide me slowly controller. In your notes, annotate what is the implication of the exploration. Does AA imply congruence or similarity in triangles?
- ✎ In your notes and here, does AA in a Euclidean triangle imply AAA in that triangle?

- ✎ Look at AAA and AAAA quadrilaterals above the video. Explore the GeoGebra by sliding and then by dragging one of the vertices in the bottom right. In your notes, sketch pictures and annotate what is the implication of the exploration, both for AAAA in a quadrilateral. Does AAAA imply congruence or similarity in quadrilaterals?
- Consider quadrilaterals DABC and EABC as related to SSASA in this diagram. What is congruent and what is not congruent?
- ✎ Sketch the counterexample in your notes and label the relevant sides and angles.
- Consider how the counterexample for SSASA leading to similar quadrilaterals is also a counterexample for reversing the order for ASASS and their subsets
- Did the counterexample we just drew show that ASSA in quadrilaterals does not lead to similarity?
- ✎ Sketch $ADCB$ and $A'DCB'$ in your notes and consider ASSA for them.
- How many angles or sides can suffice to determine congruency or similarity in a quadrilateral?
- Consider how AA similarity is applied in trigonometric definitions.
- Is triangle ABC similar to the triangle on the left?
- ✎ Write the proof and notice how some but not all items come from Euclid's Book I.
- Given the units of volume, what might we expect volume to be proportional to?
- Consider the model and what you might change it to obtain a better fit.
- Consider the two models for the bass fish. Which is better correlated to weight?
- Consider how similarity comes into play when we slide a ribbon off a box.

Euclidean and spherical perspectives interactive video

- ✎ Consider straight lines in prior educational experiences and why they are important in real life. Write down one related item you find interesting.
- What is intrinsically straight on a sphere?
- We can consider $41.885°$ N, $87.63°$ W in the area of Chicago, Illinois, USA and $41.885°$ N, $12.495°$ E in the area of Rome, Italy. They are on the same latitude. Is the non-equator latitude between Chicago and Rome intrinsically straight on the sphere?
- Read through Euclid's 5th postulate, Postulate 5, as well as Proposition 31.
- How many intrinsically straight great circle parallels can we find to a given great circle through a point off of it?
- ✎ Consider angles in prior educational experiences and why they are important in real life. Write down one related item you find interesting.
- If we drive from the North Pole on a perfectly spherical planet down to the equator and turn to travel east, what angle have we traveled?

- ✎ Consider circles in prior educational experiences and why they are important in real life. Write down one related item you find interesting.
- What is the spherical circle with center the North Pole and radius πr?
- ✎ Consider angle sums in prior educational experiences and why they are important in real life. Write down one related item you find interesting.
- ✎ In your notes, sketch the spherical triangles ABC and label the sum of the angles on them as $181.119°$ and $433.591°$ degrees
- ✎ Consider the Pythagorean theorem in prior educational experiences and why it is important in real life. Write down one related item you find interesting.
- On the sphere, if $a = 3$ and $b = 4$, then what can we say about the spherical hypotenuse?
- What would happen to the Pythagorean theorem on the sphere for tiny triangles?
- ✎ Consider squares in prior educational experiences and why they are important in real life. Write down one related item you find interesting.
- Can we form squares on a sphere consisting of four right angles (90 degrees) and four equal sides that are portions of intrinsically straight great circles?
- ✎ Consider polygons in prior educational experiences and why they are important in real life. Write down one related item you find interesting.
- ✎ Sketch a picture of a lune in your notes (the shaded wedge) and label it as a lune with 2 congruent angles and 2 congruent sides.
- For the dodecahedron (shown on the slide), how many pentagons come together at each vertex?
- Are there more, less, or the same number of spherical polyhedra compared to Euclidean polyhedra?
- Which of the following is *not* a theorem for congruence?
- ✎ Sketch the 2 triangles that satisfy SAS on a sphere but aren't congruent. Include the tick marks to show the congruent sides and included angle and the shading to show the difference in the areas of the triangles.
- ✎ Write the argument of why AA implies AAA in Euclidean geometry.
- ✎ Sketch two Euclidean triangles with AAA that are similar but not congruent.
- ✎ Sketch the 2 spherical triangles ABC and self-intersecting $AB'C'$ in your notes, with the shading too, that show AAA but not similarity
- ✎ Sketch the 2 spherical lunar triangles ABC and $AB'C$ in your notes, that show AAA but not similarity
- ✎ Consider surface area in prior educational experiences and why they are important in real life. Write down one related item you find interesting.

- ✎ In your notes, write down the formula for the surface area of a sphere as $4\pi r^2$.
- For area of a plane and surface area of a sphere, in terms of the metric system, what are the units expressible as?

Pythagorean theorem interactive video

- If the rescue squad has a 50 ft ladder and angle it to a building, 35 ft away, how many feet high will it reach?
- Review the proof of the Pythagorean theorem using AA similarity
- What is the side length of the small square?
- Multiply out and reduce.
- ✎ Next sketch the picture and write the steps and reasons in your notes.
- Consider that if we only start with a single right triangle as given, then we have underlying assumptions in creating and applying the illustration, like how do we know we even have squares?
- ✎ Read through Proposition 13 in Euclid's Elements, write it in your notes, and consider how it applies here.
- ✎ Label your figure to show this and clarify the sides opposite the 2 angles in the bottom vertex.
- ✎ Write out the argument that if we start with a triangle we can form the figure from the 周髀算經 or Zhoubi Suanjing using transformations and show that we do have squares.
- Watch the water wheel Pythagorean theorem video from the link above this video.
- ✎ Sketch the water wheel demonstration in your notes and annotate the water flow too.
- Why are the triangles congruent? *I-4 SAS, I-8 SSS, I-26 ASA or AAS
- ✎ Write Proposition 41 from Euclid's Elements in your notes and consider how it applies here.
- What goes wrong in spherical geometry?
- Watch the mathematicsonline video of Proposition 47 from the link above the video.
- ✎ Write down items related to the strengths and differences of the 3 proofs of Proposition 47: the similarity argument, the Zhoubi Suanjing argument, and I-47.

analytic geometry and metric perspectives interactive video

- ✎ In your notes, write down two points and calculate their distance by hand by using the Pythagorean metric, either in 2D or 3D, your choice.
- ✎ In your notes, write down the distance formula for the taxicab metric.

- ✎ For the coordinate geometry of the plane, the first coordinate tells us how far to the right on the positive horizontal x-axis to go. The second coordinate tells us how far up the positive vertical y-axis to go. (A negative would be in the opposite direction). Set axes and units, graph $A = (2, 5), B = (2, 1)$, and $C = (4, 1)$ and label the vertices and their coordinates.
- ✎ Graph $A' = (5, 4), B' = (3, 2)$, and $C' = (4, 1)$ and label their vertices and coordinates.
- What is the taxicab distance between $A' = (5, 4)$ and $B' = (3, 2)$?
- Compute the taxicab distances from $A = (2, 5)$ to $C = (4, 1)$ and separately from $A' = (5, 4) to C' = (4, 1)$.
- ✎ Write a proof in your notes [that SAS fails in taxicab geometry]
- Review the Euclidean proof of SAS (I-4) from the congruence and similarity 1 interactive video notes you took.
- ✎ Write in your notes and also respond here: Where does the proof of SAS (I-4) first fail in taxicab geometry?
- Open the taxicab tool from the link above the video and try it yourself.
- Compare your proof with mine.
- Compute the Pythagorean distance when the horizontal distance is 490 and the vertical distance is 764. approximately 908 feet
- ✎ In your notes, sketch the taxicab squares, label their taxicab lengths in terms of the number of blocks traveled, and the areas 9 blocks^2 versus 8 blocks^2
- Consider the images and how circumference is $2\pi f$ and ds is from the Pythagorean theorem. Plus we approximate with a flat region on an infinitesimal level
- Consider how the Euclidean metric and algebra came in to the derivation of the surface area of a surface of revolution
- Given $f(x) = \sqrt{r^2 - x^2}$, compute $f'(x)$ using chain rule
- Consider the computations.
- If we cut a spherical loaf of bread into equal width slices, which has the most crust?
- ✎ Write out this proof that each pieces has the same amount of crust and of the sphere's surface area.
- Divide the surface area of a piece cut like before by the surface area of the entire sphere. What do we obtain as a function of the width of the piece and the radius of the sphere?
- ✎ Sketch the triangle and compute $\sin(23.5°)$ in your notes [don't round]. In addition, approximately what percentage of the earth is between the tropics [do round]?

polyhedra video

- ✎ In your notes, create a different example that still satisfies these rules.

- What is $V - E + F$ of your region in the plane that satisfies the rules in the experiment?
- ✎ In your notes, write down V, E, F and $V - E + F$ for each regular polyhedra.
- Does the icosahedron have a symmetry rotation of $\frac{2\pi}{3}$?
- ✎ Consider the symmetries of the regular polyhedral and in your notes, identify a line and nontrivial angle for one of the polyhedra, including a sketch
- ✎ Write down the definitions of n as the total # of faces, k as the # of sides of one of its polygons, and p as the # of polygons that touch at a vertex. Next, internalize why $p = 3$ and $k = 4$ for a cube via considering 3 4-sided squares touching at each vertex. Then, what is p, k and n for a dodecahedron?
- How can we express V in terms of n, k, and perhaps p?
- ✎ Write down Part 1 of the proof in your notes.
- When $p = 3$ and $k = 6$ what happens to $\dfrac{2}{p} + \dfrac{2}{k}$?
- ✎ Write part 2 of the proof that there are exactly 5 regular Euclidean polyhedra in your notes
- Why does $k = 2$ work on a sphere but not the plane?
- ✎ In your notes, write down why the interior angle of a pentagon is $108°$ degrees.
- What is the interior angle of a hexagon?
- Consider the argument.

measurements and angle sum interactive video

- ✎ Sketch a picture showing a circle with a polygon inscribed and a polygon circumscribed
- ✎ Write Archimedes' argument for the area of the circle in your notes.
- ✎ Sketch the images in your notes relating to the surface area of a sphere.
- ✎ In your notes, write the usual formulas for the volume of the sphere, cone and cylinder.
- What familiar theorems have we assumed?
- ✎ In your notes, describe how we were able to drive or walk a flat Euclidean angle sum and what it showed us.
- ✎ What can we logically infer from this relationship that $a+b+c-\pi = \frac{area}{r^2}$(and write your reasoning)?
- What is the sum of the angles minus π for a triangle that goes from the North Pole to the equator, one-quarter the way around, and back up?
- Which letter relates to where we can find corresponding angles in a diagram:

- Read the bracketed summary statements of I-32, I-31, and I-29 in the Euclid's Book 1 handout.
- Sketch a pic and what axiom lets us construct the parallel?
- Write these details of the first part of the proof of I-32.
- Consider the application of CN 2 about adding congruent items, CN4 about coinciding items being congruent, and CN 1, which is basically a transitive or substitution notion.
- Write the rest of the proof of I-32.
- What first goes wrong with the proof on the sphere?
- Write the computation in your notes [for Escher's model].
- Write the computation in your notes. What is the angle sum in this triangle in M.C. Escher's hyperbolic model?

parallels 1 interactive video

- Write down as many definitions or connections to parallels as you can think of.
- What are intrinsically straight paths in spherical geometry?
- What are intrinsically straight paths in Escher's hyperbolic model?
- How many intrinsically straight parallels to a line through a point off it are there in Escher's hyperbolic model?
- Consider Euclid's 5th and guess the punchline.
- Write down the negation of Euclid's 5th.
- How many ways can we consistently fill in this game?
- Where is Euclid's 5th true?
- Where is Playfair's true?
- Think about why I-12 first and then I-11.
- Think about what is wrong and which proposition it contradicts.
- Write the proof of I-31, the existence portion of Playfair's.
- What first goes wrong with the proof of I-31 on the sphere?
- Sketch a pic as you write the details.
- Add the congruent vertical angles $\sphericalangle ADC$ and $\sphericalangle BDE$ to your picture.
- Label the pairs of sides as congruent with a single tick mark for $\overline{CD}$ and $\overline{DB}$ and a double tick mark for $\overline{AD}$ and $\overline{DE}$.

- ✎ Write the rest of the proof but not the unshown part where we would bisect $\overline{AB}$.
- ✎ Sketch a pic of a spherical triangle and E coming back down and meeting the extension of $\overline{AB}$ so that the exterior angle is not larger, even for shortest distance spherical triangles, as in the picture on the right.
- How many intrinsically straight parallels to a line through a point off it are there in spherical geometry?

parallels 2 interactive video

- ✎ Write the statements of Playfair's and Euclid's 5th in your notes.
- ✎ Sketch a picture of the givens for the proof of Euclid's 5th.
- ✎ Read the statement of I-23 in Euclid's book 1 and then add G to your picture so that $\sphericalangle GEB$ is congruent to $\sphericalangle EBC$ with G on the same side as A and D.
- Which proposition is this?
- Consider how we are applying CN2
- ✎ Write down what are the propositions we used from Euclid in this first part of the proof.
- ✎ Highlight the 4 angles in your picture.
- ✎ Write down the remaining propositions we needed to prove Playfairs + prior to I-28 $\rightarrow$ Euclid's 5th?
- ✎ Review the proof of I-31 from notes you previously took for parallels 1 and write the propositions we used.
- Consider how the givens for this part of the proof of uniqueness are the perpendicular to the perpendicular from I-31 plus a second line through P.
- ✎ Write out this proof by contradiction.
- Why aren't Euclid's 5th and Playfair's logically equivalent? Consider a geometry where one is true and the other is false.
- Consider that on the sphere any 2 great circles intersect on both sides of any transversal so Euclid's 5th is always true and Playfair's is always false because there are no great circle parallels
- Review your responses on the worksheet on hyperbolic geometry 1 for Hyperbolic Shortest Distance Paths.
- Review your responses on the worksheet on hyperbolic geometry 1 for Hyperbolic Angle Sum.
- Review your responses on the worksheet on hyperbolic geometry 1 for Hyperbolic Euclid's 5th.
- Review your responses on the worksheet on hyperbolic geometry 2 for Hyperbolic Playfair's.
- Review your responses on the worksheet on hyperbolic geometry 2 for SMSG 16.
- Review your responses on the worksheet on hyperbolic geometry 2 for the Hyperbolic Parallel Axiom
- Consider these pictures and the Hyperbolic Parallel Axiom.

- ✎ In your notes, fill in each entry as true or false in each geometry.
- Consider this quotation from Henri Poincaré, for whom the Poincare disk model of hyperbolic geometry we've been exploring is named for

parallels 3 interactive video

- Which lets us construct these perpendiculars?
- ✎ Sketch a picture and label it.
- Which lets us construct the segment?
- Which proposition shows the angles are congruent?
- Which proposition shows the triangles are congruent?
- ✎ Write the proof and add the AAS markings to your picture.
- Consider what first goes wrong in the non-Euclidean geometries?
- ✎ Write down in your notes what first goes wrong with the proof in the non-Euclidean geometries: I-29 in hyperbolic and no parallels on the sphere.
- Which is true for shortest hyperbolic paths?
- Which is true for the existence portion of Playfair's proof?
- Is the existence portion of Playfair's true in spherical geometry?
- Is the existence portion of Playfair's in hyperbolic geometry?
- Why doesn't the Pythagorean theorem hold in hyperbolic?
- Consider the 3 models and how they can reveal behavior about intrinsically straight paths, parallels and angle sums.
- What would the 'problem' be if the real universe is spherical?
- What geometry did Lobachevsky's experiment indicate?
- What geometry did Kirshner's experiments seem to indicate?
- What geometry did the density experiments indicate?
- How do polyhedra relate to possibilities for the geometry of our universe?

projective geometry interactive video

- Consider this argument that Playfair's implies cuts.
- ✎ Sketch a diagram of the existence part of Playfair's in your notes. Include m, P off it, the perpendicular, and p shown as parallel

- Consider this argument that Cuts implies Playfair's
- What happens to Proclus's assumption in hyperbolic geometry?
- Review the proof of I-32 from the measurements and angle sum interactive video and consider what happens to the exterior angle in hyperbolic and spherical geometry and how this impacts the angle sum.
- ✎ In your notes, write down what happens to the exterior angle in hyperbolic and spherical geometry and how this impacts the angle sum.
- How do the infinite planes intersect?
- What happens to Desargues when corresponding sides of a triangle are parallel?
- What happens to Desargues in hyperbolic geometry?
- What happens to Desargues for non-generate spherical triangles?
- ✎ In your notes, write down how could you explain division to a child.
- Consider that spaces that locally look like the plane are higher dimensional versions of a flat cylinder, Möbius band, donut, and Klein bottle, plus the plane itself.
- ✎ Write and respond to: what is projective geometry?
- Does SAS hold in projective geometry?
- Consider that Arthur Cayley famously said that projective geometry is all of geometry
- ✎ Write down which you find most compelling about why Euclid wrote the fifth postulate the way he did?