Analytic Geometry and Metric Perspectives 1
Dr. Sarah's MAT 3610: Introduction to Geometry

Goals:
- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate that they seem to apply in a wide variety of examples.
- Proof Considerations
  I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
 I can compare and contrast multiple geometric perspectives.

Welcoming Environment: Actively listen to others and encourage everyone to participate and try to help each other! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and ask me questions during group work time as well as when I bring us back together:

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

Euclidean and Taxicab Metrics

2. The Euclidean metric measures distance as the crow flies. Consider why the base sides have the lengths shown. Then assume the Pythagorean theorem holds and apply it to solve for the hypotenuse as the usual distance formula between 2 points: $d((x_1, y_1), (x_2, y_2))$ where d is the distance between the points. At the same time, replace the absolute values with parenthesis when you square the distances—and show this.
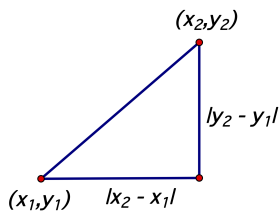
3. Distance is measured differently on other spaces like the sphere or where buildings are in the way. One example is on a city grid where taxicabs can only travel on horizontal and vertical roads. Here is an image of part of Charlotte from Google Earth:

   Go to
   `https://nrich.maths.org/14444`
   and scroll down past the intro video to the Treasure Hunt. Input a starting coordinate for the taxicab treasure hunt where $x \in (0, 12), y \in (0, 12)$. *Test coords* and you'll see the number of steps (total horizontal and vertical distance summed) to the treasure. Use the clues to find the treasure! How many guesses did it take? A different version is at `https://www.geogebra.org/m/qaf9pswg`

4. Discuss the treasure hunt with your group. Who found the treasure in the least number of tries?

5. Open
   `https://www.geogebra.org/geometry/enku7tgq`
   Notice that the coordinates of A are $(-6, 4)$. Keep A at that location, but drag E to $(-5, 3)$ and thus compute the taxicab distance from $A = (-6, 4)$ to $E = (-5, 3)$ in GeoGebra. List it here.

6. Drag E to find points on and off the grid that are a distance of 2 from $(-6, 4)$ in the GeoGebra exploration. Use this to sketch here the full taxicab circle of radius 2 about $(-6, 4)$—it isn't a Euclidean circle!

7. Consider the treasure hunt and the GeoGebra exploration that shows the taxicab distance as the sum of the vertical and horizontal distances. What is the general formula for $d\,\text{taxi}((x_1, y_1), (x_2, y_2))$, the taxicab distance between points? You'll need absolute values here!

8. Consider if the treasure hunt game game gives you a distance of 4 steps from a point with coordinates $(6, 7)$. How does this connect to a taxicab circle, i.e. what is the center and radius of a related taxicab circle?

   center: radius:

9. Write out an analytic geometry proof that the Euclidean metric is what you solved for above in #2, i.e. take the reasoning you did in #2 and turn it into a paragraph argument. Be sure to explain how you are applying the Pythagorean theorem. I've started it for you:

   Proof: To derive the distance formula for the Euclidean metric between two points at coordinates (x_1, y_1) and (x_2, y_2), form a right triangle with base...

10. Trade your proof and provide each other with feedback.

11. **Help each other and PDF responses to ASULearn**: If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit this, continue reviewing and solidifying or discuss upcoming class work. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.