Worksheet on Desargues' Theorem

Dr. Sarah's MAT 3610: Introduction to Geometry

Goals:
- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate that they seem to apply in a wide variety of examples.
- Proof Considerations
  I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
  I can compare and contrast multiple geometric perspectives.

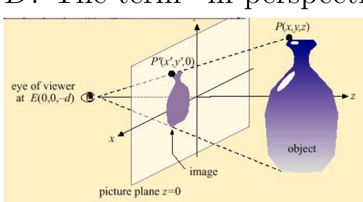
Welcoming Environment: Actively listen to others and encourage everyone to participate and try to help each other! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and ask me questions during group work time as well as when I bring us back together:

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

2. Desargues' Theorem, named for French mathematician Girard Desargues, is a famous result in projective geometry: when two triangles are in perspective then the 3 intersections of the corresponding sides are collinear. Open up GeoGebra Geometry for a Euclidean exploration:

   - Use the **Polygon** tool to create triangle ABC. You will have to click back on A to complete the triangle.
   - Create a **Point** D outside the triangle and line **Segment**s $\overline{AD}$, $\overline{BD}$, and $\overline{CD}$.
   - Use the **Polygon** tool to create triangle EFG with E on the segment $\overline{AD}$, F on the segment $\overline{BD}$, and G on the segment $\overline{CD}$ (see the image on the right below).
   - **Move** E to be sure it is constrained to the segment $\overline{AD}$, and similar for F and segment $\overline{BD}$, and G and segment $\overline{CD}$. Notice that two triangles are in perspective with respect to the point D. The term "in perspective" arises from perspective drawing, like in the left image below.



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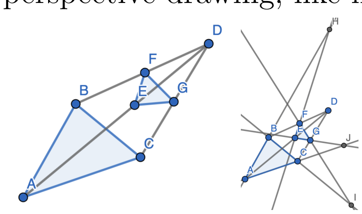


   - Next, create the **Line**s $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$.
   - If needed, Zoom out or move some points, so that you can see their intersection (if they are parallel, move one of the points so they are not). Then **Intersect** the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ at point H.
   - Similarly, create the intersection of lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{FG}$, point I, and the intersection of lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{EG}$ at point J.

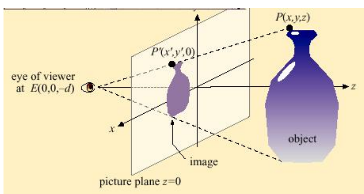
   Do the 3 intersections of the corresponding sides seem to be collinear as in Desargues' theorem? (if not, try again)

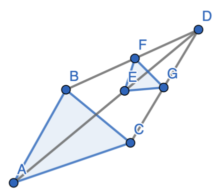
3. What happens if we move the point D. What else changes and what remains the same?

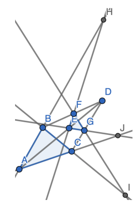
4. What happens if we move the point E? What else changes and what remains the same?

5. What happens to Desargues' theorem about the 3 intersections being collinear if one or more sides of the triangle EFG are parallel to the sides of triangle ABC?

6. To show that Desargues' theorem holds in Euclidean geometry when there are 3 intersections, imagine lifting D and triangle EFG into 3-space, kind of like the picture with the vase.







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Now if $\triangle ABC$ and $\triangle EFG$ lifted are in two different infinite planes that are not parallel, then what is the intersection of those two planes? If you are stuck, consider intersecting your hands. Separately consider the ceiling and wall intersecting. And, if you have taken linear algebra, consider the set of solutions to that linear system that form the intersections. They all provide the same response. What is the intersection? **circle one**: point line plane other

7. Discuss with your group that would this show that H, I, and J must be on the same line once we project back to 2-space and **circle**: discussed

8. Open up
`https://www.geogebra.org/m/akwvszgu`
Set up Desargues theorem like you did previously in Euclidean geometry. In the order you will need them, you'll want to use **Hyperbolic Segment** (since there is no Hyperbolic polygon tool), **Point**, **Hyperbolic Segment**, **Hyperbolic Line**, and **Intersection of Hyperbolic Lines**. Aside from **Point** and **Move**, be sure to use the hyperbolic tools under the wrench symbols. What happens to Desargues' theorem in hyperbolic geometry?

9. Open
`https://www.geogebra.org/m/gqsbq9md`
and explore this by considering what you first see and then drag L and then M. If needed, add to your response in #8.

10. **Help each other and PDF responses to ASULearn**: If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit this, continue reviewing and solidifying or discuss upcoming class work. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.