Worksheet on Hyperbolic Geometry Part 2

Dr. Sarah's MAT 3610: Introduction to Geometry

Physical Geometry Manipulative: hyperbolic crochet model

Goals:
- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate that they seem to apply in a wide variety of examples.
- Proof Considerations
  I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
  I can compare and contrast multiple geometric perspectives.

Welcoming Environment: Actively listen to others and encourage everyone to participate and try to help each other! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and ask me questions during group work time as well as when I bring us back together:

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

Playfair's Postulate

2. Playfair's axiom says: given a line and a point not on it, exactly one line parallel to the given line can be drawn through the point. To show we can create a path that doesn't intersect, we'll create a hyperbolic sketch that follows along with the Euclidean proof, so open up a new version of `https://www.geogebra.org/m/akwvszgu`.

   –Under the first wrench, use the **Hyperbolic Line** to create a line going through two points that are inside the Poincaré disk (not on the boundary). Notice that the program will show four points, two in the disc A and B and two on the boundary at infinity.

   –Under the second wrench, use the **Hyperbolic Drop Perpendicular** and select A, B, and a point off the line through them (E). Notice that you will have a perpendicular to $\overline{AB}$.

   –Next, under the second wrench, select **Hyperbolic Perpendicular at Point** and choose E and any another point on the perpendicular aside from the intersection with $\overline{AB}$. You have created the perpendicular to the perpendicular, which never intersects $\overline{AB}$, so it is parallel.

   This part of the Euclidean proof only required up to I-27, which did not require Euclid's 5th postulate, so it is not surprising that the construction still works in this model of hyperbolic geometry. Sketch a diagram of your construction.

3. Connect E with B using a **Hyperbolic Segment** under the first wrench. Then measure the alternate interior angles of the parallels cut by $\overline{EB}$ using **Hyperbolic Angle**, including $\angle ABE$. Be careful to measure the alternate interior angles related to $\overline{EB}$—you may have to reverse the order if you obtain the exterior angle rather than the interior angle. Do the alternate interior angles seem to be equal?

4. Sketch a picture that illustrates your response and identify the components.

5. Review I-29. Does the picture indicate that it fails?

SMSG Postulate 16

6. Write down the form of the parallel postulate given in the SMSG Axioms as SMSG Postulate 16 and also identify the two givens here.

7. Is SMSG Postulate 16 true on the sphere? Sketch a related picture.

8. Is SMSG Postulate 16 true in Euclidean geometry? Sketch a related picture.

9. Is SMSG Postulate 16 true in hyperbolic geometry? Sketch a related picture.

What Goes Wrong with the Euclidean Proof of the Sum of the Angles

10. Review our Euclidean proof that the sum of the angles in a triangle is 180° (I-32) from the measurements and angle sum interactive video. What goes wrong in the Euclidean proof for hyperbolic geometry? Use what we did in the Playfair's Postulate explorations above as you analyze the proof in order to help you answer this question.

11. Show a hyperbolic sketch of the given and beginning of the proof as well as the very first place that the proof fails, and annotate what goes wrong.

Hyperbolic Parallel Axiom

12. The Hyperbolic Parallel Axiom states that if l is a hyperbolic line and P is a point not on l, then there exist exactly two noncollinear hyperbolic halflines which do not intersect l and such that a third hyperbolic halfline intersects l if and only if it is between the other two (and doesn't intersect it otherwise). Make sense of this axiom by creating a hyperbolic sketch that illustrates it in `https://www.geogebra.org/m/akwvszgu`.
    Use the **Hyperbolic Line** tool to create the line l. Then, use the usual point tool to create a point P off the line (it might be called E). Next select the **Hyperbolic Segment** tool to create a hyperbolic segment through P that **does intersect** l—put your second endpoint on the other side of l. Sketch a picture.

13. Then, move the endpoint (it might be called F) close to the boundary but stay inside the disk. Drag it all the way around, still close to the boundary, to see when the segment will intersect l and when it won't. Try to find the two halflines "which do not intersect l and such that a third hyperbolic halfline intersects l if and only if it is between the other two (and doesn't intersect it otherwise)." Sketch a picture that includes l and the two halflines.

14. **Help each other and PDF responses to ASULearn**: If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit this, continue reviewing and solidifying or discuss upcoming class work. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.