Worksheet on Pythagorean theorem

Dr. Sarah's MAT 3610: Introduction to Geometry

goals:

- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate they seem to apply in a wide variety of examples.
- Proof Considerations
  I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
  I can compare and contrast multiple geometric perspectives.

Pythagorean Theorem Visualizations

1. Explore `https://www.geogebra.org/m/fYDFzQ5N` by dragging the slider k. If you label the big square opposite C as having side c, the square opposite A as having side a and the square opposite B as having side b, then what does this visualization seem to show about $a^2 + b^2$ versus c^2?

2. Explore `https://www.geogebra.org/m/cSvQ3svX` as follows:
   - First go to the slider point on the b^2 square and drag it. Notice that is a shear.
   - Then slide the new point that arose on the other side of the b^2 square and slide it along the dotted line (another shear).
   - Then slide the next new point in the opposite corner of the b^2 square into the c^2 square (a rotation).
   - Repeat with the a^2 square.
   - Do these transformations preserve the areas of the b^2 square and a^2 square?

Pappus Extension

3. Instead of squares on the outside of the sides of a right triangle, construct equilateral triangles on a right triangle and measure their areas as follows (the tools are in italics):
   - Using a new GeoGebra Geometry, construct a *Segment* $\overline{AB}$
   - Construct a *Perpendicular Line* through A to $\overline{AB}$.
   - Construct a *Point* on the perpendicular, called C.
   - Construct *Segment*s $\overline{AC}$ and the hypotenuse $\overline{BC}$.
   - You can *Show/Hide Object* to hide the perpendicular line so that you have only the right triangle. Then drag the point A to ensure you seem to have a right triangle.
   - Under Polygons, use *Regular Polygon* to construct an equilateral triangle—select the segment C then B and then put in 3 for the vertices. If the triangle is on the inside rather than the outside of $\triangle ABC$ then undo and select B and C in the reverse order.
   - Measure its *Area*. Make note of what GeoGebra calls it (poly1?).
   - Similarly, construct equilateral triangles on the outside of the other two sides and measure their areas.

- Using the calculator, input poly2 + poly3 and hit return (if those aren't the names in the areas of the equilateral triangles on the base sides, use their names)
- Compare the result with the area of the first equilateral triangle. What do you notice?
- Drag point A to show this relationship seems to hold in a wide variety of examples.

4. In your notes or below, roughly sketch the construction you created in GeoGebra to discover the relationship between the areas of the equilateral triangles sitting on the sides of the right triangle.
5. In your notes or below, roughly sketch a separate equilateral triangle with a side of length c, and sketch an altitude h.
6. The altitude breaks the original equilateral triangle into two triangles. What are its sides and why? Solve for the altitude too and show work.
7. Using your last response, what is the area of the equilateral triangle with side length c?
8. Using substitution, what is the area of the equilateral triangle with side length a? b?
9. Write the sum of the areas of equilateral triangles of side lengths a and b.
10. Factor the constant and then apply the Pythagorean theorem to show the relationship between the areas of the equilateral triangle holds.
11. Using 7–10, write out a paragraph proof relating the sum of the areas of equilateral triangles of sides a and b on the bases of a right triangle to the equilateral triangle of side c on the hypotenuse, and identify underlying assumptions.

Other Polygons

12. Construct a regular pentagon on the outsides of a right triangle, measure the pentagon areas, and check if a Pythagorean-type of relationship seems to hold.
13. Construct another regular polygon on the outsides of a right triangle, measure the polygon areas, and check if a Pythagorean-type of relationship seems to hold. How many sides did you use?

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14. Try to fit the puzzle pieces into a square.
15. Sketch the completed puzzle in your notes or below. If you label the hypotenuse as c, the longest base as a and the short base as b, then what does the completed puzzle seem to show?