Worksheet on Pythagorean Theorem 2

Dr. Sarah's MAT 3610: Introduction to Geometry

Goals:
- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate that they seem to apply in a wide variety of examples.
- Proof Considerations
  I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
  I can compare and contrast multiple geometric perspectives.

Welcoming Environment: Actively listen to others and encourage everyone to participate and try to help each other! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and ask me questions during group work time as well as when I bring us back together:

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

Applications of the Pythagorean Theorem

2. Instead of squares on the outside of the sides of a right triangle, construct equilateral triangles on a right triangle and measure their areas as follows (the tools are in italics):
   a) Using a new GeoGebra Geometry, construct a *Segment* $\overline{AB}$
   b) Construct a *Perpendicular Line* through A to $\overline{AB}$.
   c) Construct a *Point* on the perpendicular, called C.
   d) Construct *Segment* $\overline{AC}$
   e) Construct the hypotenuse $\overline{BC}$ using *Segment*.
   f) You can *Show/Hide Object* to hide the longer perpendicular line so that you have only the right triangle—it hides after you choose another tool. Then drag the point A to ensure you seem to have a right triangle.
   g) Under Polygons, use *Regular Polygon* to construct an equilateral triangle—select the segment via the points C then B and then put in 3 for the vertices. If the triangle is on the inside rather than the outside of $\triangle ABC$ then undo and select B and C in the reverse order.
   h) Measure its *Area*. Make note of what GeoGebra calls it (poly1?).
   i) Similarly, construct equilateral triangles on the outside of the other two sides and measure their areas.
   j) Using the calculator, at the bottom, input poly2 + poly3 and hit return (if those aren't the names in the areas of the equilateral triangles on the base sides, use their names)
   k) Compare the result with the area of the first equilateral triangle. **What do you notice—what relationship do you seem to have discovered?**
   l) Drag point A to show this relationship seems to hold in a wide variety of examples.

3. Roughly sketch the construction you created in GeoGebra to discover the relationship between the areas of the equilateral triangles sitting on the sides of the right triangle.

4. Roughly sketch one equilateral triangle here with a side of length c, and sketch an altitude h.

5. The altitude breaks the original equilateral triangle into two congruent triangles via SAS or similar. Sketch one of these smaller triangles and label all 3 of its sides.

6. Assume the Pythagorean theorem holds. Solve for the altitude h in terms of the side of length c and show work.

7. Using your last response and the formula for the area of a triangle, what is the area of the equilateral triangle with side length c in #4 in terms of c?

8. Using direct substitution, what is the area of an equilateral triangle with side length a? with side length b?

9. Write the sum of the areas of equilateral triangles of side lengths a and b.

10. Factor the constant and then apply the Pythagorean theorem to show the relationship between the areas of the equilateral triangle holds.

11. Assuming #6–#8 as true, write out a paragraph proof relating the sum of the areas of equilateral triangles of sides a and b on the bases of a right triangle to the equilateral triangle of side c on the hypotenuse. Start with the three equilateral triangles on the right triangle as given. Consider how I started and ended the proof and fill in the middle. Be sure to show how you are applying the Pythagorean theorem like in #9 and #10 and include the words "Pythagorean theorem" in your proof.

    Proof: Assume we have a right triangle with bases a and b and hypotenuse c. Also assume we have three equilateral triangles on its sides, an equilateral triangle with sides a on the base of side a of the right triangle and pointing to the outside of the right triangle, and similar for the others. We will show that the sum of the areas of the triangles on the bases equals the area of the triangle on the hypotenuse. The areas of the triangles on the bases sum to...

    ...So, the sum of the areas of the equilateral triangles of sides a and b on the bases of the right triangle equals the area of the equilateral triangle of side c on the hypotenuse of the right triangle. Q.E.D.

Pythagorean Exploration with Other Polygons

12. Construct a regular pentagon on the outsides of a right triangle, measure the pentagon areas, and check if a Pythagorean-type of relationship seems to hold. Summarize what you discovered.

13. Construct another regular polygon with at least 6 sides on the outsides of a right triangle, measure the polygon areas, and check if a Pythagorean-type of relationship seems to hold. Report back.

14. Specify how many sides did you use?

15. **Help each other and PDF responses to ASULearn**: If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit this, continue reviewing and solidifying or discuss upcoming class work. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.