Worksheet on Spherical Perspectives

Dr. Sarah's MAT 3610: Introduction to Geometry

Physical Geometry Manipulatives: sphere, string and masking tape

Goals:

- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate that they seem to apply in a wide variety of examples.
- Proof Considerations
  I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
  I can compare and contrast multiple geometric perspectives.

Welcoming Environment: Actively listen to others and encourage everyone to participate and try to help each other! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and ask me questions during group work time as well as when I bring us back together:

1. **Building Community**: What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

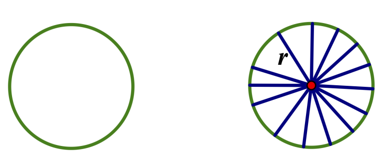
Euclid's Postulates on the Sphere

2. Given two points A and B on the surface of a ball, what is the intrinsically straight path between them, the path that a toy car would take without turning its steering wheel, masking tape would follow without buckling, or string pulled tightly would fall on? These aren't extrinsically straight from the point of view of someone outside the ball, but they are intrinsically straight for those living on the space. Provide the name from the video.

3. Read Euclid's Postulate 1. By using your last response as the intrinsically straight lines on the sphere—we have symmetry that could satisfy Definition 4's "lies evenly with the points on itself"— sketch 2 points A and B on the sphere, and an intrinsically straight line between them, demonstrating axiom 1.

4. Read Euclid's Postulate 2. Can we continue along straight lines on the sphere, wrapping back around as needed?

5. Euclid's Postulate 3. In Euclidean geometry, a circle is the set of points equidistant from a center with a fixed radius r. Use string, with its length as the radius, and start from a point P as the north pole (NP) to look at the set of points equidistant from P. What would the spherical circle of center P and radius the string length look like? Include a sketch of the sphere, the NP point, and the spherical circle.



Euclidean circle r creating it

6. If we were to vary the length of the string, consider an unusual spherical circle we could obtain. Keep increasing the length of the string in your mind. If you are stuck, let me know. Sketch and explain.

7. Euclid's Postulate 4 is true on the sphere. The way we measure angles on the surface of the sphere is the change in direction—the steering a car would make, or the angle that the tangent lines to the great circles show, like if we held out sticks (the tangents) as we were turning. For example, if we head south along a longitude from the north pole to the equator and turn east to travel on the equator, what angle do we have?

8. Euclid's first 27 propositions employ what comes before them, with the exception of Euclid's 5th postulate, which is not used until $I - 28$. We will explore Euclid's 5th later, but it is true on the sphere as stated. Based on the postulates only, as they are explicitly stated, should Euclid's propositions hold on the sphere?

Proposition 1: Equilateral Triangle Construction and Proof Revisited

9. Open
`https://www.geogebra.org/m/jpzbqbtw`
and sketch a picture of what you see there initially, including the labels of A and B.

10. How many equilateral spherical triangles are shown?

11. Next, drag A and B far apart. Don't stop—keep dragging them apart! Sketch a related picture, including the labels of A and B.

12. In the axiomatic systems and constructions 2 interactive video, I asked you to write out the proof of $I - 1$ in your notes, so review that Euclidean proof. What happens to the equilateral triangle construction from the proof of $I - I$ of *Euclid's Elements* on the sphere for large segments—does it always produce equilateral triangles on $\overline{AB}$?

13. If not, identify the first underlying assumption from the proof that fails and explain.

Proposition 4: SAS Revisited

14. Using Euclid's definition of intrinsically straight paths—but not necessarily shortest distance paths—sketch two spherical triangles that satisfy the conditions of SAS on the sphere but are not congruent. The unusual one was in the Euclidean and spherical perspectives interactive video.

15. Review the Euclidean proof of SAS ($I - 4$) from the congruence and similarity 1 interactive video notes you took. In the video we looked at some underlying assumptions and limitations of the Euclidean proof. Where does the Euclidean proof of SAS first go wrong? Explain.

16. Reword Postulate 1 so that the underlying assumption that is used in the proof of $I - 4$ is explicitly stated.

17. **Help each other and PDF responses to ASULearn**: If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit this, continue reviewing and solidifying or discuss upcoming class work. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.