

Worksheet on Spherical Angle Sum and AAA

Dr. Sarah's MAT 3610: Introduction to Geometry

Physical Geometry Manipulatives: sphere and masking tape

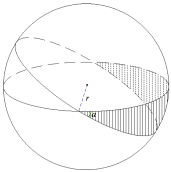
Goals: • Geometric Perspectives

I can compare and contrast multiple geometric perspectives.

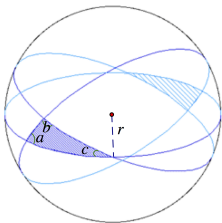
Welcoming Environment: Actively listen to others and encourage everyone to participate and try to help each other! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and ask me questions during group work time as well as when I bring us back together:

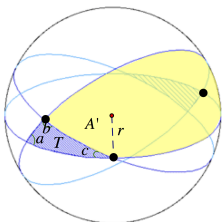
1. **Building Community:** What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.
2. To explore angle sum on the sphere of spherical triangles that aren't self-intersecting nor have any 180° angles, first create a lune on the sphere using masking tape that doesn't buckle to create two great circles that go all the way around the ball. Next, the area of a lune of angle a radians = $\frac{a}{2\pi} \times \text{surface area of sphere}$, so multiply $\frac{a}{2\pi} \times 4\pi r^2$ and reduce.



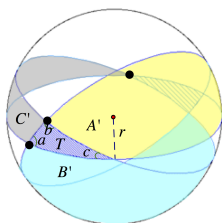
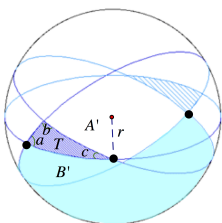
3. Use masking tape to create a third great circle all the way around in order to make a spherical triangle T as in the following picture and label on the tape angles a , b , and c . Then consider Euclid's *Elements* Book I Definitions 19 and 20, with straight lines there as intrinsically straight great circles here. Do these satisfy Euclid's definition of a trilateral figure?



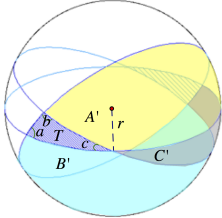
4. Opposite angle a across the side is another region on the sphere—call it A' . Notice that $T + A'$ together make up a lune of angle a . So, what the area of $T + A'$ in terms of the angle a and the radius r ?



5. Similarly, across b we have B' and across c we have C' . What is the area of $T + B'$? $T + C'$?



6. From your prior responses, what is $3T + A' + B' + C' = (T + A') + (T + B') + (T + C')$ in terms of the three angles and the radius r ?
7. There is another copy of C' on the sphere opposite or antipodal to the one above. Visualize this on the sphere to see the congruency and that this copy fills in the space between A' and B' . Using this opposite or antipodal copy, what portion of the sphere do $T + A' + B' + C'$ fill up? And what is its area?



8. Take the equation in #6 $3T + A' + B' + C' =$ (your response to #6) and subtract the equation in #7 $T + A' + B' + C' =$ (your response to #7) so that you now have a formula for
- $$3T + A' + B' + C'$$
- $$- (T + A' + B' + C')$$
- $$= 2T.$$
- Show work for your response to #6 – your response to #7 and then set it equal to $2T$.
9. Divide both sides by 2 and factor out r^2 . What is T , the area of the spherical triangle, now:
- $$T = r^2(\quad)?$$
10. T has a positive area and r^2 is positive, so the other term must be positive too. What does this tell us about the sum of the angles as compared to π ?
11. As another application, if we have regular spherical triangles (that aren't self-intersecting nor have any 180° angles) then what can we say about the areas of two spherical triangles satisfying AAA? Apply #9 and explain.
12. Does AAA on the sphere for these (non-degenerate) triangles provide congruence? Explain why or why not.
13. Sketch a counterexample to AAA on the sphere from the Euclidean and spherical perspectives interactive video—of two self-intersecting triangles or two lunar triangles.
14. **Help each other and PDF responses to ASULearn:** If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. Then submit this, continue reviewing and solidifying or discuss upcoming class work. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.