Worksheet on Analytic/Metric Geometry

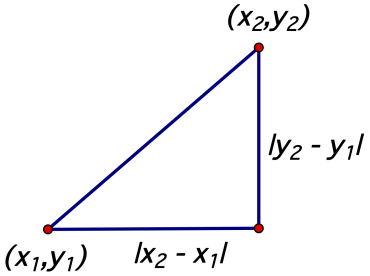
Dr. Sarah's MAT 3610: Introduction to Geometry

goals:

- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate they seem to apply in a wide variety of examples.
- Proof Considerations
  I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
  I can compare and contrast multiple geometric perspectives.

Metrics

1. With the Euclidean metric, the distance between points, d, is defined as $d((x_1, y_1), (x_2, y_2)) = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$. Can you see why from applying the Pythagorean theorem to the picture?



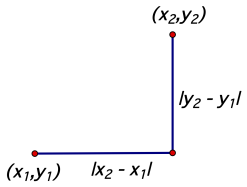
2. Assume the Pythagorean theorem holds and write out a proof that the Euclidean metric is $d((x_1, y_1), (x_2, y_2)) = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$.

The Euclidean metric measures distance as the crow flies. Distance is measured differently on other spaces like the sphere or where buildings are in the way. One example is on a city grid where taxicabs can only travel on horizontal and vertical roads. Here is an image of part of Charlotte from Google Earth:

3. Read through
`https://archive.learner.org/teacherslab/math/geometry/shape/taxicab/index.html`
How is the the shortest distance between two points defined for taxicab geometry on this page?

4. Next, hide the treasure and play one or two games to see the taxicab metric at work. How many moves and blocks did it take to win?

5. What is the distance between points in the taxicab metric $d((x_1, y_1), (x_2, y_2))$?



Taxicab Circles

6. Open `https://www.geogebra.org/geometry/enku7tgq`. Notice that the coordinates of A are $(-6, 4)$. Keep A at that location, but drag E to investigate the set of all points that are distance 2 away from A in the taxicab metric (i.e. the taxicab circle of radius 2 with center $(-6, 4)$).

7. Sketch the full taxicab circle of radius 2 about $(-6, 4)$ (it isn't a Euclidean circle!).

8. What is the taxicab circumference/perimeter of this taxicab circle?

9. What is the diameter across?

10. π is defined historically as the ratio of the circumference to the diameter of a circle, so compute this. Is taxicab π the same as what you obtain using the Euclidean metric?

Taxicab Equidistant Points

11. In the same GeoGebra document `https://www.geogebra.org/geometry/enku7tgq`, insert points at $H = (0, 0)$ and $I = (3, 3)$

12. Move A to a point you think is equidistant from H and I in the taxicab metric (i.e. the same taxicab distance away from both points).

13. Test out your guess by first moving E to H to see the taxicab distance and then to I to see the taxicab distance. If they aren't the same, try a different location for A.

14. Find all of the points that are equidistant from H and I. Once you have some points add them to the sketch on the board. There are more than you might initially think!

15. Sketch all the equidistant points.

16. In a town having perfect square blocks and equally spaced streets running north and south as well as east and west, two police stations are to be located at $H = (0, 0)$ and $I = (3, 3)$. The town officials want to divide the town into two precincts—Precinct 1 served by Station H and Precinct 2 served by Station I. What are the real-life issues that would go into deciding how the boundary should be drawn?