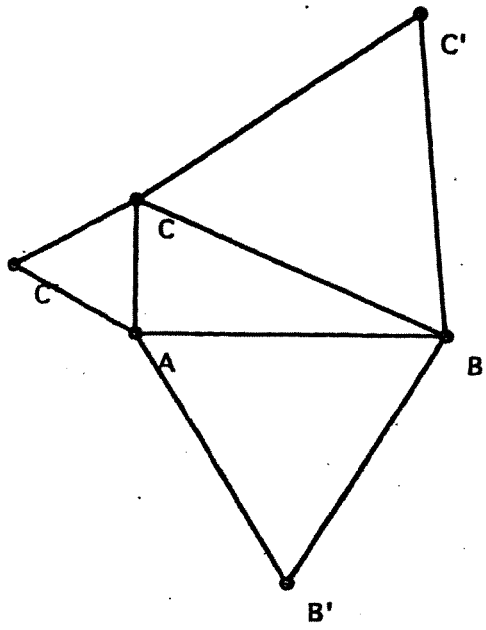
The Theorem of Pappus

A historical note: Pappus of Alexandria lived in the fourth century AD and is considered the last of the great classical mathematicians. His theorem on right triangles is actually very general: it says that if the hypotenuse is one side in any polygon, and if we then construct polygons on the legs similar to the one on the legs, then the area of the polygon on the hypotenuse will be the sum of the areas of the other two polygons. In modern books, it's only the application to regular polygons that we see.

1. Construct a segment *AB*.
2. Construct a perpendicular to *AB* at *A*.
3. Construct a point *C* on this perpendicular, and construct segment *AC*. (You can then hide the rest of the perpendicular line.)
4. On each side of right triangle *ABC*, construct an equilateral triangle



5. To measure the area of a triangle, we must first construct its interior. Select the three vertices of one of the triangles and then choose "Triangle Interior" from the Construct menu. Do the same for the other two triangles.
6. For each triangle, first click in its interior and then choose "Area" from the Measure menu.
7. Use the calculator to determine if the area of the largest triangle is in fact the sum of the areas of the two smaller ones. Does it work?
8. Let's see if we can explain the theorem. If an equilateral triangle has sides of length *s*, how long is its altitude?

9. What is the formula for the area of an equilateral triangle?

10. How can we use the Theorem of Pythagoras to explain the Theorem of Pappus for equilateral triangles?

If you have time, you might try repeating the construction with regular pentagons instead of equilateral triangles. (see the other side)

Pythagoras Plus

Use Sketchpad to explore further with the Pythagorean Theorem. Remember how we used squares on the sides of a right triangle to show the Pythagorean Theorem? Recall that in a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. Do you think this relationship will hold for polygons besides squares? Test the idea.

1. Open a new sketch and construct a right triangle. Next, construct a regular polygon (a polygon with all sides congruent and all angles congruent) of some kind so that the hypotenuse is one side of the polygon. Construct the same kind of regular polygon on each leg of your triangle, using the leg as one side of your polygon. Construct the polygon interiors.
2. Use the Measure menu and the Sketchpad calculator to find and display the area of your three polygons. Show whether the area of the figure constructed on side **a** plus the area of the figure constructed on side **b** equals the area of the figure constructed on hypotenuse **c** in your sketch. Print out a copy with all measures and your conclusions displayed. Turn it in to your teacher.