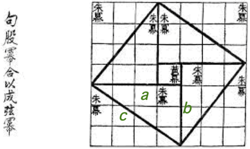
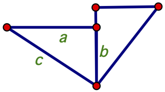
# Why is the Pythagorean Theorem True?

Dissection Arguments: Try to fit the puzzle pieces into a square

# Why is the Pythagorean Theorem True?




周髀算經 or *Zhoubi Suanjing*

Use the labels and the puzzle...

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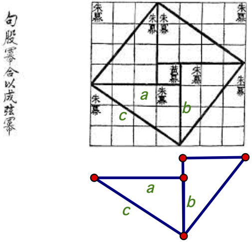


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Use the labels and the puzzle...

large square has side *c*

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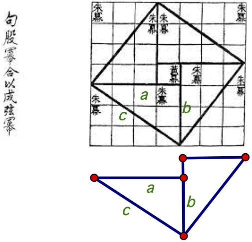


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Use the labels and the puzzle...
large square has side $c$
small square has side

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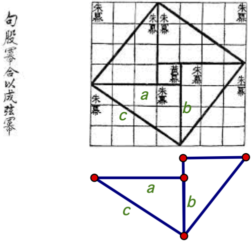


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Use the labels and the puzzle...
large square has side $c$
small square has side $a - b$

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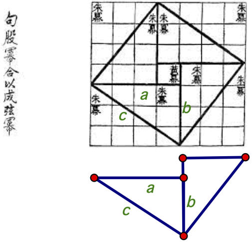


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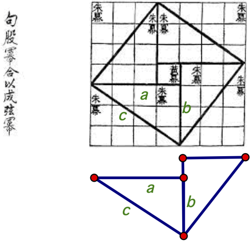
Use the labels and the puzzle...
large square has side $c$
small square has side $a - b$
area of the large square = small square + 4 triangles
$c^2 =$

# Why is the Pythagorean Theorem True?

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Use the labels and the puzzle...

large square has side $c$

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area of the large square = small square + 4 triangles

$c^2 = (a - b)^2 + 4(\frac{ab}{2})$

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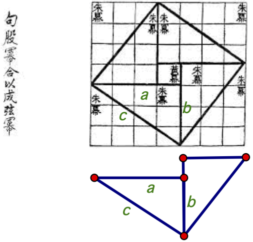


周髀算經 or *Zhoubi Suanjing*

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$$c^2 = (a-b)^2 + 4\left(\frac{ab}{2}\right)$$

$$c^2 = a^2 - 2ab + b^2 + 2ab$$

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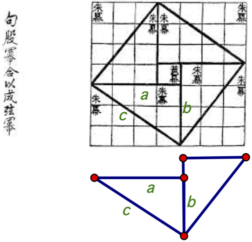


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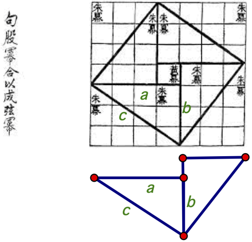



周髀算經 or *Zhoubi Suanjing*

Use the labels and the puzzle...

large square has side $c$

small square has side $a - b$

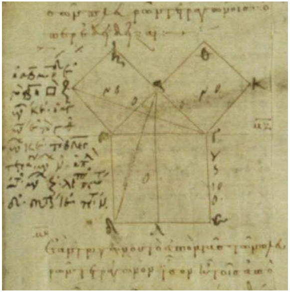
area of the large square = small square + 4 triangles

$$c^2 = (a-b)^2 + 4(\frac{ab}{2})$$
$$c^2 = a^2 - 2ab + b^2 + 2ab$$
$$c^2 = a^2 + b^2$$

Proof: Where should we start? Why do we have squares?

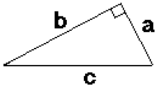
# Why is the Pythagorean Theorem True?

https://www.youtube.com/watch?v=CAkMUdeB06o

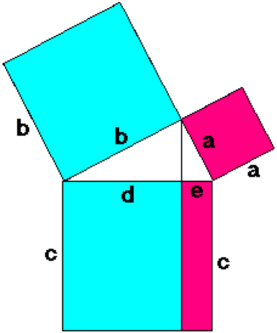


Translation of *Euclid's Elements*

**To prove:** $a^2 + b^2 = c^2$



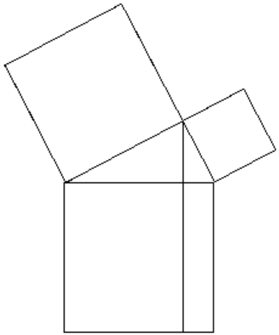

by Dave Lantz at Colgate




To prove: $a^2 + b^2 = c^2$

In fact, $a^2 = ce$
and $b^2 = cd$

by Dave Lantz at Colgate



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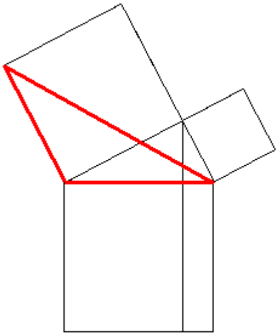

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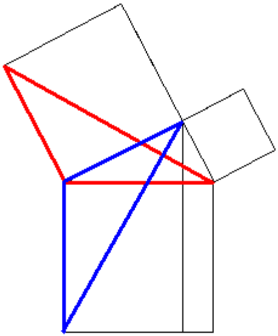

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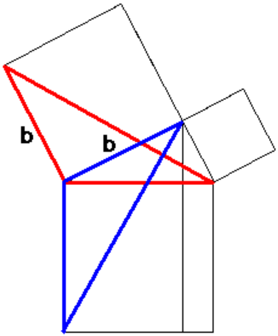


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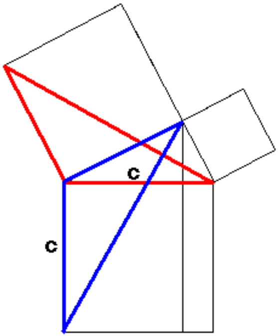


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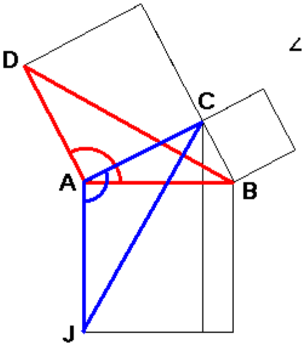


$\angle DAB$

$= \angle CAB$ + a right angle

$= \angle CAJ$

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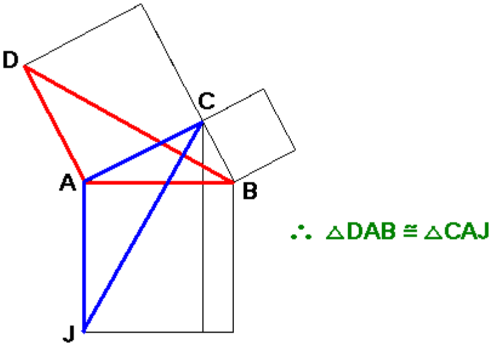


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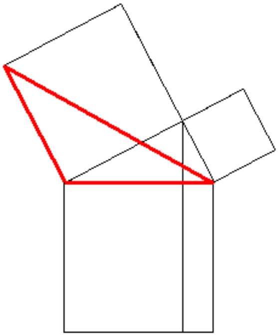

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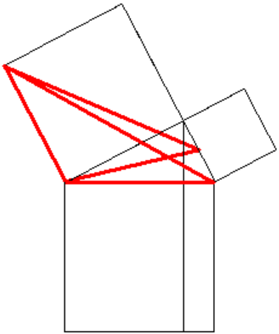

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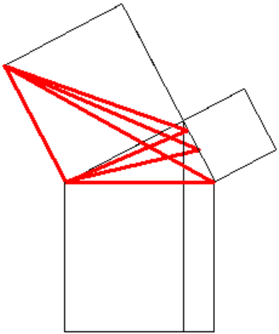

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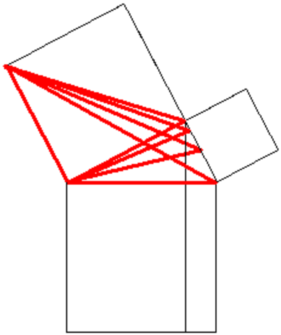

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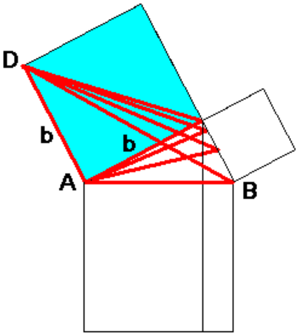
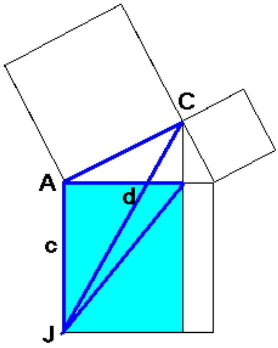



$$\text{area}(\triangle DAB) = \frac{1}{2} b^2$$

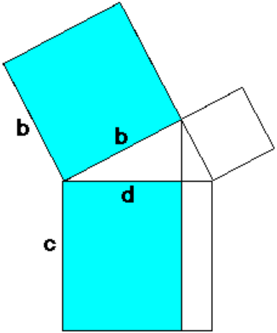
by Dave Lantz at Colgate




Similarly,

area( $\triangle CAJ$ ) = $\frac{1}{2}$ cd

by Dave Lantz at Colgate



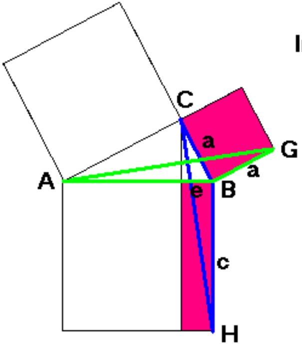

$\text{area}(\triangle DAB) = \frac{1}{2} b^2$

Similarly,

$\text{area}(\triangle CAJ) = \frac{1}{2} cd$

$\therefore\ b^2 = cd$

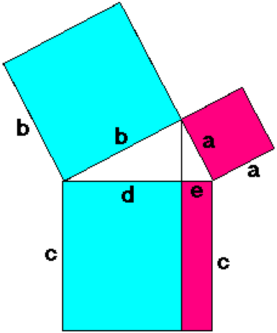
by Dave Lantz at Colgate




In the same way,

$$a^2 = 2\,\text{area}(\triangle GAB)$$
$$= 2\,\text{area}(\triangle CHB)$$
$$= ce$$

by Dave Lantz at Colgate




**Therefore:**

$$
\begin{aligned}
b^2 + a^2 &= cd + ce \\
&= c\,(d+e) \\
&= c^2
\end{aligned}
$$

**QED!**

by Dave Lantz at Colgate

https://www.youtube.com/watch?v=nxi8gV6_50o
[9:15] turn on annotations

- What are aspects you like about this video from mathmaticsonline? What are aspects that could be improved?

`https://www.youtube.com/watch?v=nxi8gV6_50o`
[9:15] turn on annotations

- What are aspects you like about this video from mathmaticsonline? What are aspects that could be improved?
- axiom 2: common notion 2 (If equals are added to equals, then the wholes are equal.)
- shear transformation
- equal areas, not congruent

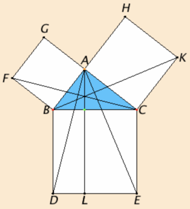
# Proposition 47

*In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.*

Let *ABC* be a right-angled triangle having the angle *BAC* right.

I say that the square on *BC* equals the sum of the squares on *BA* and *AC*.

Describe the square *BDEC* on *BC*, and the squares *GB* and *HC* on *BA* and *AC*. Draw *AL* through *A* parallel to either *BD* or *CE*, and join *AD* and *FC*. I.46 I.31, I.Post.1




Since each of the angles *BAC* and *BAG* is right, it follows that with a straight line *BA*, and at the point *A* on it, the two straight lines *AC* and *AG* not lying on the same side make the adjacent angles equal to two right angles, therefore *CA* is in a straight line with *AG*. I.Def.22 I.14

For the same reason *BA* is also in a straight line with *AH*.

Since the angle *DBC* equals the angle *FBA*, for each is right, add the angle *ABC* to each, therefore the whole angle *DBA* equals the whole angle *FBC*. I.Def.22 I.Post.4 C.N.2

Since *DB* equals *BC*, and *FB* equals *BA*, the two sides *AB* and *BD* equal the two sides *FB* and *BC* respectively, and the angle *ABD* equals the angle *FBC*, therefore the base *AD* equals the base *FC*, and the triangle *ABD* equals the triangle *FBC*. I.Def.22 I.4

Now the parallelogram *BL* is double the triangle *ABD*, for they have the same base *BD* and are in the same parallels *BD* and *AL*. And the square *GB* is double the triangle *FBC*, for they again have the same base *FB* and are in the same parallels *FB* and *GC*.

Therefore the parallelogram *BL* also equals the square *GB*. I.41

Similarly, if *AE* and *BK* are joined, the parallelogram *CL* can also be proved equal to the square *HC*. Therefore the whole square *BDEC* equals the sum of the two squares *GB* and *HC*.

And the square *BDEC* is described on *BC*, and the squares *GB* and *HC* on *BA* and *AC*. C.N.2

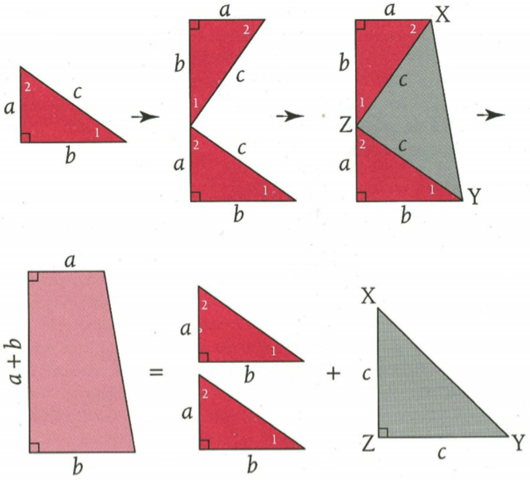
Therefore the square on *BC* equals the sum of the squares on *BA* and *AC*.

Therefore *in right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle*..

Q.E.D.

David E. Joyce `https://mathcs.clarku.edu/~djoyce/java/elements/bookI/propI47.html`

## parallelogram equals square (area)



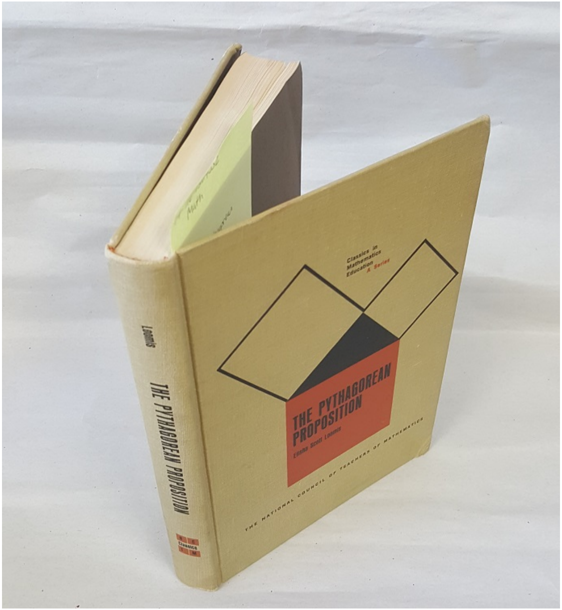

$$\tfrac{1}{2}(a+b)(a+b) = 2(\tfrac{1}{2}ab) + \tfrac{1}{2}c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$\textit{Therefore, } a^2 + b^2 = c^2$$

https://www.maa.org/press/periodicals/convergence/mathematical-treasure-james-a-garfields-proof-of-the-pythagorean-theorem

# Elisha Scott Loomis




367 proofs of the Pythagorean theorem

# Where Else is Pythagorean Theorem Historically?

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Babylonian cuneiform Plimpton 322

cuneiform

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Babylonian cuneiform Plimpton 322

cuneiform

Baudhayana sutras (Vedic Sanskrit texts) also predate Pythagoras.

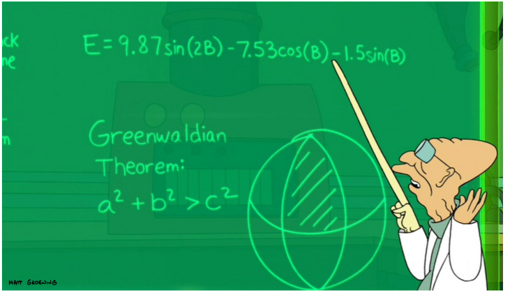
दीर्घचतुरश्रस्याक्ष्णया रज्जुः पार्श्वमानी तिर्यग् मानी च यत् पृथग् भूते कुरुतस्तदुभयं करोति ॥

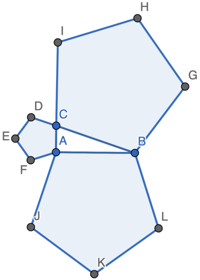
> A rope stretched along the length of the diagonal produces an area which the vertical and horizontal sides make together.

Egypt 3, 4, 5 right triangles

# Extensions of the Pythagorean Theorem

- non-Euclidean geometry
- squares to other regular polygons on the right triangle
- squares to other algebraic powers

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*The Wizard of Oz*

*$pringfield: The Simpsons*

**Homer**: The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.

- What is wrong with Homer's version of the theorem?

# *The Scarecrow's Theorem?*

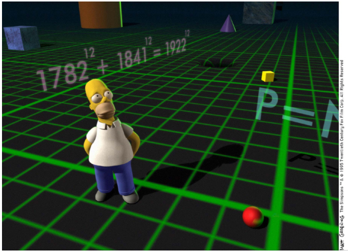
*The Wizard of Oz*

*$pringfield: The Simpsons*

**Homer**: The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.

- Is this statement true for any triangle? Find an example of a triangle with sides $a$, $b$, and $c$ so that $\sqrt{a} + \sqrt{b} = \sqrt{c}$ or explain why it is not possible to find such a triangle.

# Squares to Power of 12

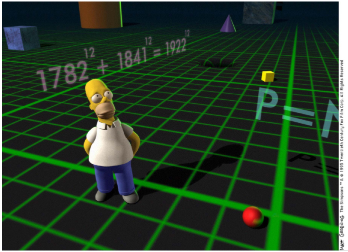



*The Simpsons: Treehouse of Horror VI, Homer*$^3$

$$1782^{12} + 1841^{12} = 1922^{12}$$

- Is this equation true or false? Why?

# Squares to Power of 12




*The Simpsons: Treehouse of Horror VI, Homer*[3]

$$1782^{12} + 1841^{12} = 1922^{12}$$

- Is this equation true or false? Why?
- On a calculator, I type (1782ˆ12 + 1841ˆ12) ˆ(1/12) ENTER and obtain 1922. Resolve the apparent conflict.

## Project 4: Concept Development of Course Topics

- analytic geometry (with coordinates/metric)
- area and area measurements
- axiomatic systems for Euclidean geometry (Euclidean geometry synthetically, without coordinates)
- geometric constructions (including IGS and dynamic geometry)
- modeling with geometry
- parallel postulates
- perimeter and circumference and their measurements
- polyhedra (three-dimensional objects)
- right triangles
- similarity
- geometric transformations
- volume and volume measurements

Mathematician David Henderson explains that: A proof...

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**is a communication**—when we prove something we are not done until we can communicate it to others and the nature of this communication, of course, depends on the community to which one is communicating and is thus in part a social phenomenon.

**is convincing**—a proof “works” when it convinces others. Of course some people become convinced too easily so we are more confident in the proof if it convinces someone who was originally a skeptic. Also, a proof that convinces me may not convince you or my students.

**answers–Why?**– The proof should explain something that the hearer of the proof wants to have explained. I think most people in mathematics have had the experience of logically following a proof step by step but are still dissatisfied because it did not answer questions of the sort: “Why is it true?” “Where did it come from?” “How did you see it?” “What does it mean?”

`https://drive.google.com/file/d/1trwDtAxDwA7x95tLR5Jr5Ni-NIdwz1bk/view?usp=`