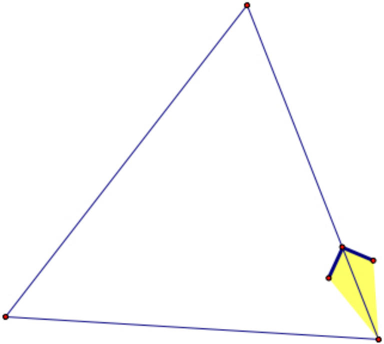
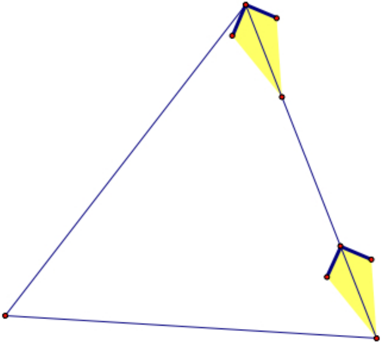
Angle Sum in Various Geometries

- Lay out a triangle with masking tape
- Pick a vertex to begin your triangle walk. Note the vertex and which way you are facing.



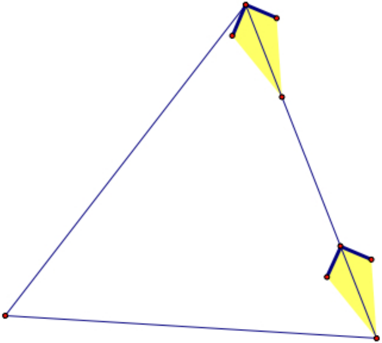
Walking a Euclidean Angle Sum

- Start walking along your triangle, keeping the center of your body on the boundary of the triangle.



Walking a Euclidean Angle Sum

- When you get to a turn (one of the angles of the triangle), turn your body so that it sweeps the interior angle of the triangle (careful!). You may be walking backwards for a time.



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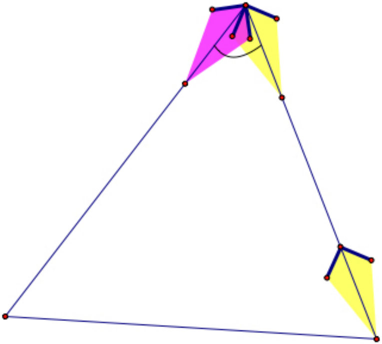

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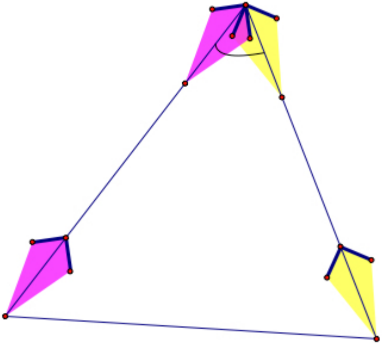

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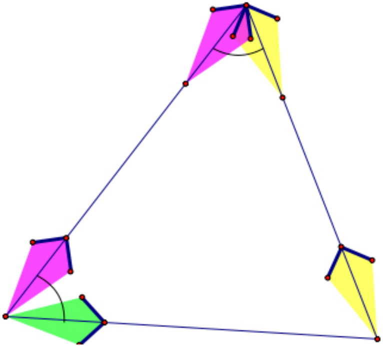

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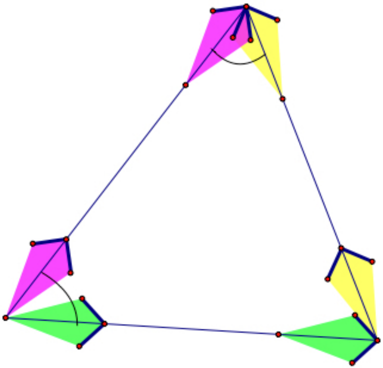
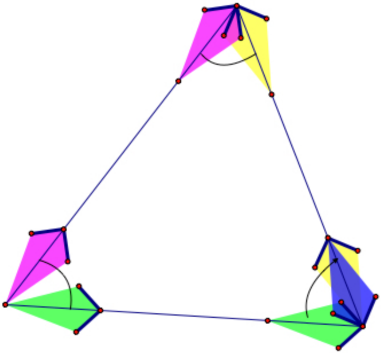


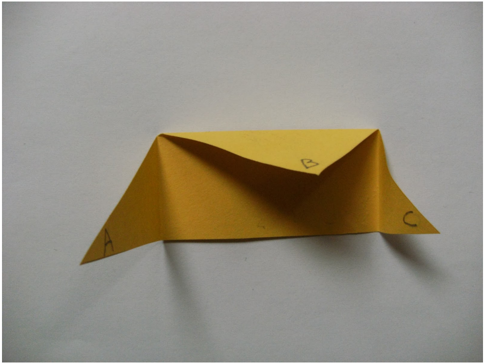
Walking a Euclidean Angle Sum

- Sweep out the last interior angle to finish your angle sum walk.
- The change in direction in your body from start to finish is the sum of the angles in this triangle.



Folding an Angle Sum Extrinsically

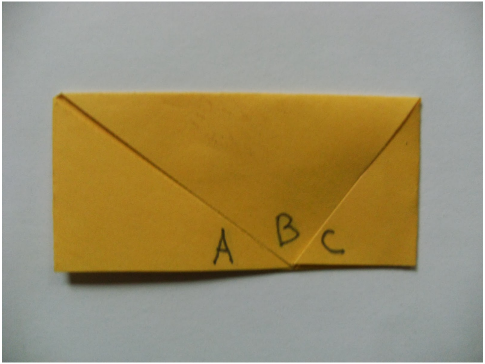
- Rip a triangle from paper.
- Fold one angle to bring it down to the base by using a fold parallel to the base.
- Fold the other angles in



http://mathonthemckenzie.blogspot.com/2013/12/180.html

Folding an Angle Sum Extrinsically

- Notice the angles fit to take up the entire space along the base and this gives us the angle sum.



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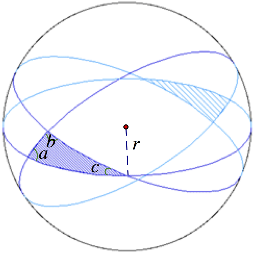
Sum of the Angles in a Triangle on a Sphere

`http://cs.appstate.edu/~sjg/class/3610/beachball.pdf`



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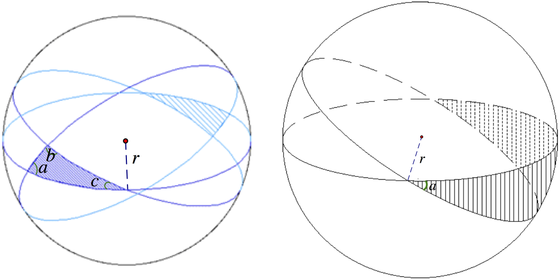

area of lune of angle *a* radians=

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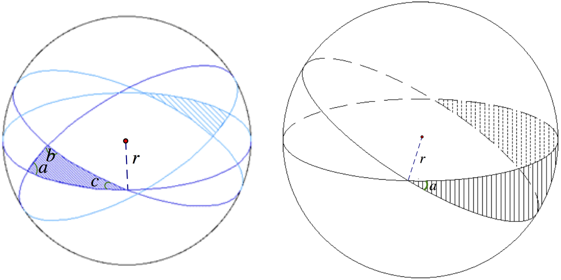

area of lune of angle a radians= $\frac{a}{2\pi} \times$ surface area of sphere

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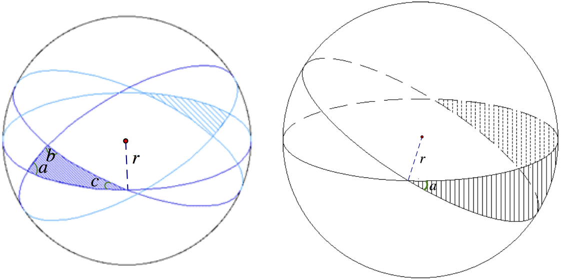

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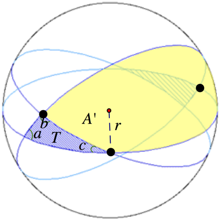

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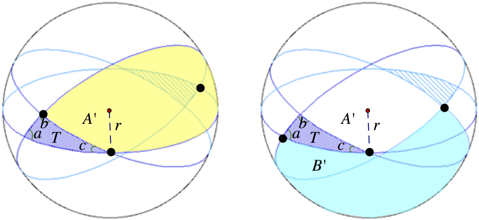

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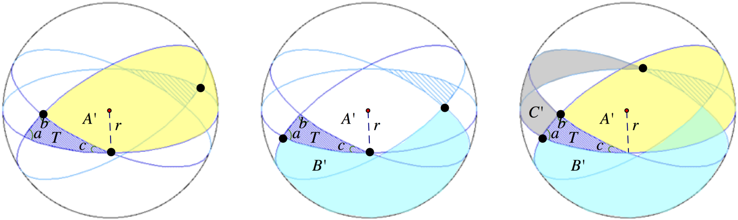

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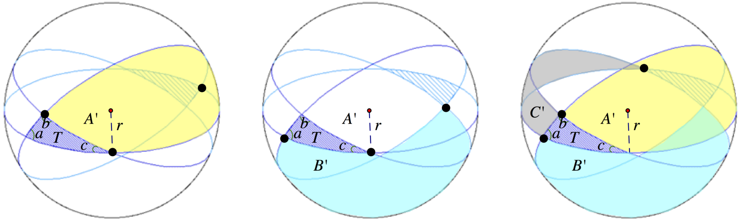

$T + A' = 2ar^2$ $\quad$ $T + B' = 2br^2$ $\quad$ $T + C' = 2cr^2$

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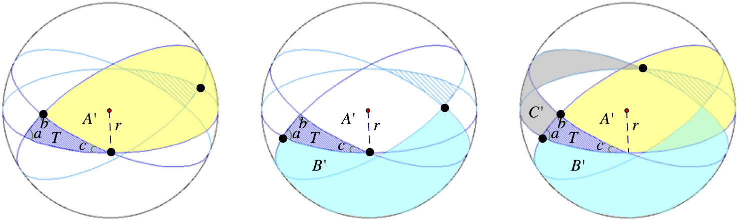

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$$3T + A' + B' + C' = 2ar^2 + 2br^2 + 2cr^2$$

Sum of the Angles in a Triangle on a Sphere

equation 1: $3T + A' + B' + C' = 2ar^2 + 2br^2 + 2cr^2$

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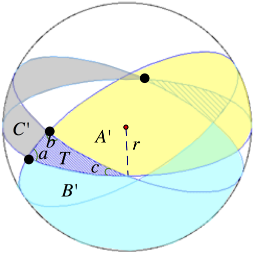

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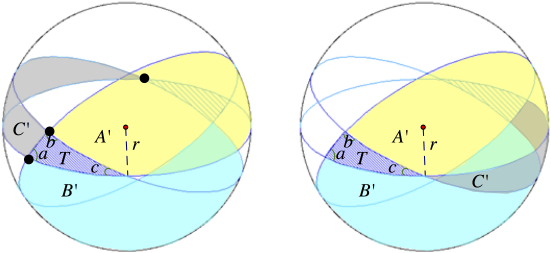

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Sum of the Angles in a Triangle on a Sphere

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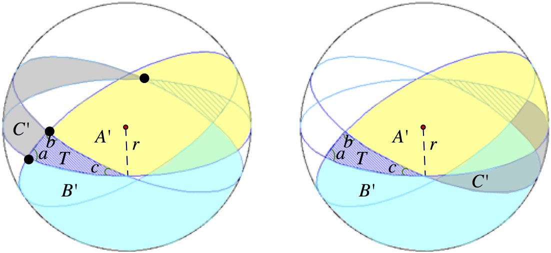

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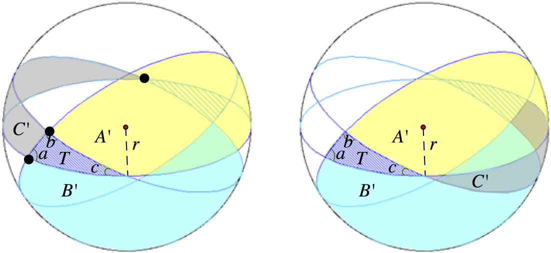

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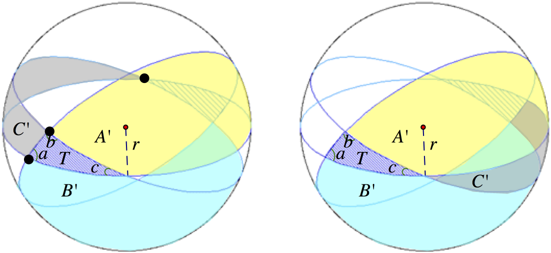
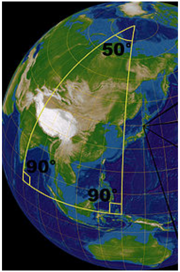
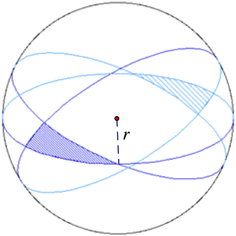


equation 2: $T + A' + B' + C' =$ hemisphere $= \frac{4\pi r^2}{2} = 2\pi r^2$
equation 1 $-$ equation 2: $2T = 2r^2(a + b + c - \pi)$
area of the triangle $= r^2$(sum of the angles $- \pi$)

Detecting the Sum of the Angles in an Earth Triangle

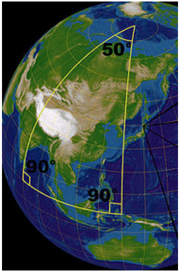


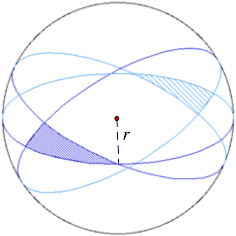


selected a subset of the image

$$\text{sum of the angles} - \pi = \frac{\text{area of the triangle}}{r^2}$$

Detecting the Sum of the Angles in an Earth Triangle

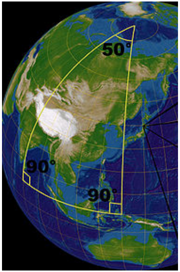


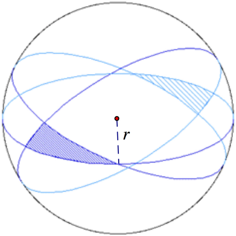


$$\text{sum of the angles} - \pi = \frac{\text{area of the triangle}}{r^2}$$

$$\frac{1}{3959^2} \approx 6.38 \times 10^{-8}$$

Detecting the Sum of the Angles in an Earth Triangle





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$$\frac{1}{3959^2} \approx 6.38\times 10^{-8} \qquad \frac{82277}{3959^2} \approx 0.005$$

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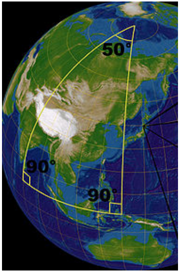

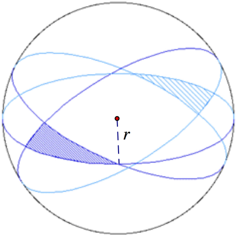

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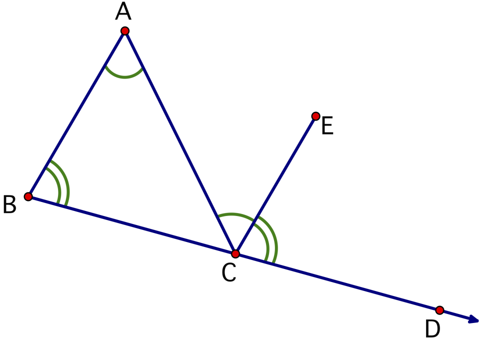
$$\frac{1}{3959^2} \approx 6.38\times 10^{-8} \qquad \frac{82277}{3959^2} \approx 0.005 \qquad \frac{196{,}000{,}000/8}{3959^2} \approx 1.57$$

Euclidean Proof of I-32

Let ABC be a triangle. To show the angle sum is 2 right angles, extend BC by Postulate 2 and let D be a point on it so C is between B and D. Next construct $\overline{CE}$ parallel to $\overline{AB}$ through C by

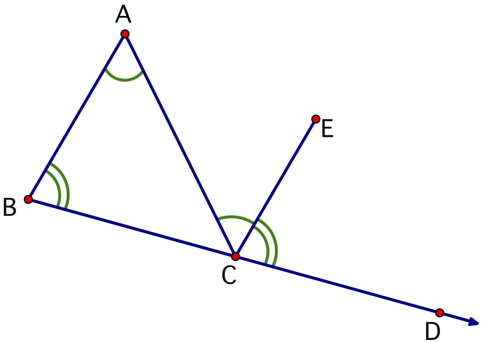
Euclidean Proof of I-32

Let ABC be a triangle. To show the angle sum is 2 right angles, extend BC by Postulate 2 and let D be a point on it so C is between B and D. Next construct $\overline{CE}$ parallel to $\overline{AB}$ through C by I-31.



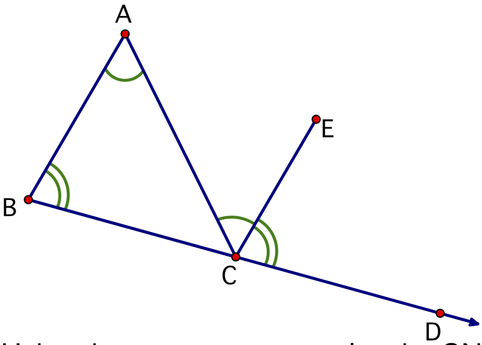
Notice that alternate interior angles $\sphericalangle BAC \cong \sphericalangle ACE$ by I-29 since the transversal $\overline{AC}$ cuts the parallels $\overline{AB}$ & $\overline{CE}$. Similarly by I-29, the corresponding $\sphericalangle ABC \cong \sphericalangle ECD$.

Euclidean Proof of I-32 continued



Using these congruent angles, by CN2, CN4, and CN 1, $\sphericalangle BAC + \sphericalangle ABC \cong \sphericalangle ACE + \sphericalangle ECD \cong \sphericalangle ACD$.

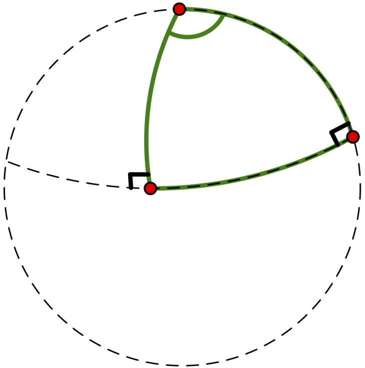
Euclidean Proof of I-32 continued



Using these congruent angles, by CN2, CN4, and CN 1, $\sphericalangle BAC + \sphericalangle ABC \cong \sphericalangle ACE + \sphericalangle ECD \cong \sphericalangle ACD$. Thus $\sphericalangle ACB + \sphericalangle ACD \cong \sphericalangle ACB + \sphericalangle BAC + \sphericalangle ABC$ by CN2. In addition, we know that $\sphericalangle ACB + \sphericalangle ACD$ is 2 right angles by I-13, so the interior angles of triangle ABC are also 2 right angles by CN1.

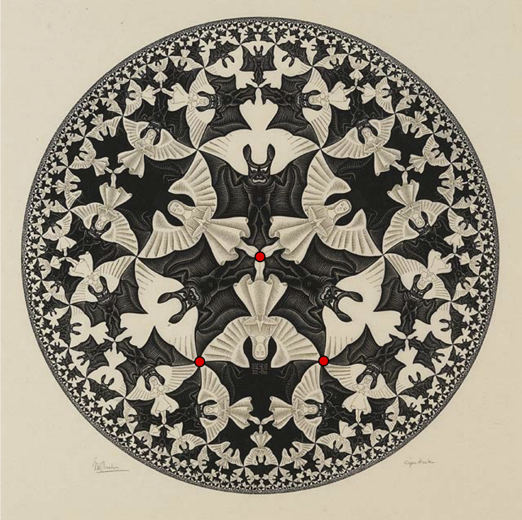
I-32 on the Sphere

$$\text{sum of the angles} - \pi = \frac{\text{area of the triangle}}{r^2}$$



- What first goes wrong with the Euclidean proof of I-32 on the sphere?

I-32 in Hyperbolic Geometry



Circle Limit 4: Heaven and Hell by M.C. Escher, 1960