Geometric Modeling Using the Sphere and Similarity

1. Open `https://www.geogebra.org/m/jpzbqbtw` and drag A and B far apart.

2. What happens to the equilateral triangle construction of I-I of *Euclid's Elements* on the sphere for large segments?

3. In the interactive video from the 2nd day of class, I asked you to write out the proof of I-1, so review that Euclidean proof of I-I. Does it always work on the sphere? If yes, just say yes, but if not, identify the first underlying assumption from the proof that fails.

Athletes compete in overall sports competitions, but they vary in height and weight, so the question is how to equitably compensate for differences in events like a weight-lifting portion.

4. Download the Excel file geomodel.xls from the webpage, which presents data from the winners of the 1976 Olympics by their weight lifting class.

5. Perhaps we could subtract the body weight of the athlete from their lift? This suggests that lift grows linearly in weight, which we can test out using Excel regression. So we will test to see whether
$$\frac{\text{lift}}{\text{weight}} = \text{constant slope, i.e. whether lift} \propto \text{weight.}$$
To do so, we'll create a linear regression plot:

   - Click on the letters B and C above the relevant columns, using the command key, so that both columns are highlighted.

   - Under Insert, look for a *Scatter* or *Marked Scatter* chart.

   - Once the chart appears, control click on one of the points on the Chart, so that the menu is brought up which lets you *Add Trendline*.

   - Click on *Display R-squared value on chart.* R^2 measures the y-values distances via a sum of squares as variation in the dependent variable explained by linearity and the closer to 100% the stronger the relationship.

   –What is the r^2 value?

   –What are the strengths and weaknesses of this model?

The error for each point, is the vertical distance the point is from the line.

The best fit line, is the one which will make the combined area of all the error squares, as small as possible.

`http://math.maine121.org/welcome/chapter-5/`
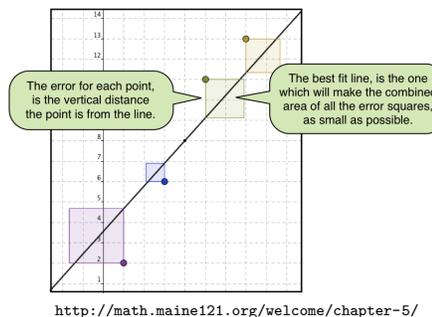
6. Researchers suggest instead that muscle strength may be proportional to cross sectional area. This would suggest that lift $\propto$ muscle strength $\propto$ area $\propto$ length^2. We can relate this to weight that is in the Excel file as follows: weight $=$ volume $\times$ density $\propto$ volume (if we assume a constant average density) $\propto$ length^3. So muscle strength $\propto$ length^2 and weight $\propto$ length^3 (so weight$^{\frac{1}{3}} \propto$ length).

   –Use this to derive: muscle strength $\propto$ weight^x (hint: the power x will be fractional).

   Once you have your power (i.e. model), test it in Excel. Click on C. Under Insert, release on Columns, to insert a new column before the Total lift column. The blank column should be column C, so Click on C2. Type

   $= \text{b2}^\wedge(x)$ (where you fill in x with your fraction).

Click back on C2 and go to the bottom right corner, where the thick plus turns into a thin plus, and then click and pull down (Excel will fill down your formula). Next create the regression plot with your new column and the lift.

–What is the new r^2 value?

–Is this model any better than the first model?

–Are there any other models you can think of using?

7. As you read the following image, sketch and label a model, and also fill in the details and reasons throughout using *Euclid's Elements* Book I, trigonometry, and the triangle similarity theorems and definitions from class.

Let ABC be a triangle with right angle at C. Its legs have lengths a and b and the length of the hypotenuse is c.

Apply the scaling factor $\frac{1}{c}$ to create a similar triangle $A'B'C'$ with legs $\frac{a}{c}$ and $\frac{b}{c}$ and hypotenuse 1.

When the Pythagorean Theorem is used on the new triangle, we have

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1^2 = 1$$

In right triangle trigonometry, we define the sine and cosine functions as follows. Consider an acute angle θ in triangle ABC. The sine of θ is the ratio created by taking the length of the opposite side and dividing by the length of the hypotenuse. In our triangle, I've arbitrarily placed θ at vertex A, and so

$$\sin(\theta) = \frac{a}{c}$$

Because ABC and $A'B'C'$ are similar, the corresponding angles are congruent. Thus, we may also write:

$$\sin(\theta) = \frac{\frac{a}{c}}{1} = \frac{a}{c}$$

Note that the sine is dependent only on the angle and not the size of the triangle.

Similarly, the cosine of θ is the ratio created by taking the length of the adjacent side and dividing by the length of the hypotenuse: $\cos(\theta) = \frac{b}{c}$. (To confirm the independence of the angle, consider $A'B'C'$ where $\cos(\theta) = \frac{\frac{b}{c}}{1} = \frac{b}{c}$.)

Now let's go back to $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$ and "substitute equals for equals" to arrive at the Pythagorean Identity for Trigonometry.

$$\left(\sin(\theta)\right)^2 + \left(\cos(\theta)\right)^2 = 1$$

This image is a lesson plan from an unknown author