

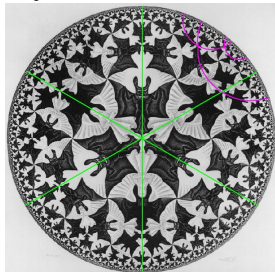
## Euclidean proof of I-32.

Discuss what goes wrong with the proof of I-32 on the sphere.

Escher's representation of hyperbolic geometry



<http://www.malinc.se/noneuclidean/images/triangleSum.svg>



## *What are various ideas that relate to parallel?*

Write down as many definitions, ideas and concepts that relate to the meaning or visualization of parallel.

**Parallel lines have  
so much in common**

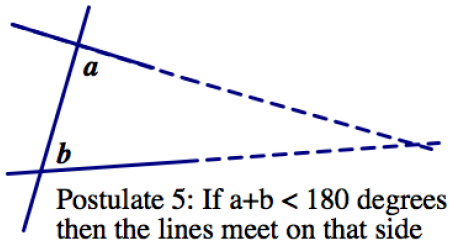


**it's a shame they'll  
never meet**



## Euclid's Elements 5th Postulate

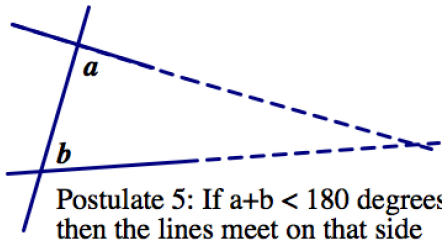
If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side...



Guess the punchline!

## Negation of Euclid's Elements 5th Postulate?

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## Consistency $\nRightarrow$ Uniqueness

Wile E. Coyote

1		1

Axiom 1) Each square is a number or a mine.

Axiom 2) A numbered square represents the number of neighboring mines in the blocks immediately above, below, left, right, or diagonally touching (or a subset of those if a block is on a boundary)

How many consistent games can you find that satisfy the initial board plus the axioms?

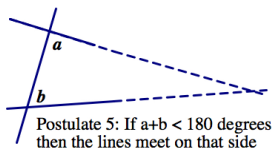
## Euclid's Elements Postulates on Plane and Sphere

Euclid's 5th:

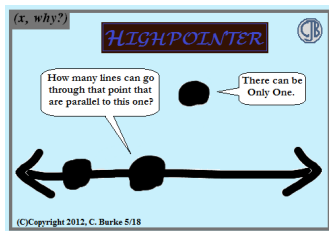
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Playfair's:

Given a line and a point off that line there is exactly 1 parallel to the line through the point.



Guess the punchline!

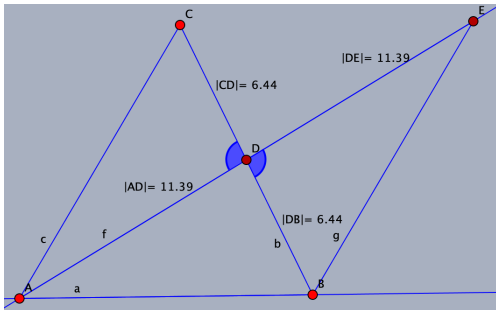


## *Existence Portion of Playfair's Postulate*

- Create a parallel—what Euclidean propositions are we using?

## *Existence Portion of Playfair's Postulate*

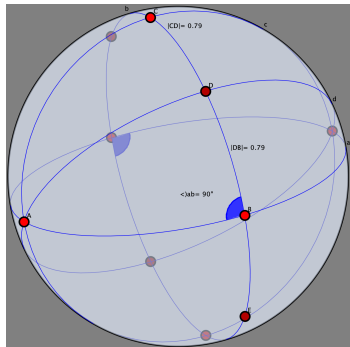
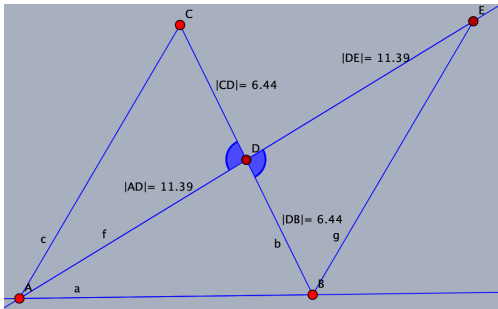
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- Why is the perpendicular to the perpendicular parallel in Euclidean geometry but not in spherical geometry?





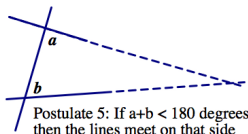
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## Existence Portion of Playfair's Postulate

- Create a parallel—what Euclidean propositions are we using?
- Why is the perpendicular to the perpendicular parallel in Euclidean geometry but not in spherical geometry?
- Euclid's 5th Postulate is vacuously true on the sphere so unlike what is listed on the web and in some books, the statements are different. We will prove: Euclid's 5th Postulate plus Euclid's other axioms before I-31 prove Playfair's (underlying assumptions like for SAS!)
- We will also investigate parallels in hyperbolic geometry.



Guess the punchline!

