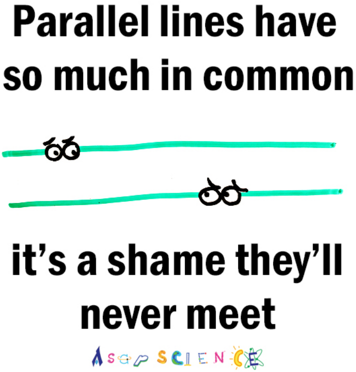
What are various ideas that relate to parallel?

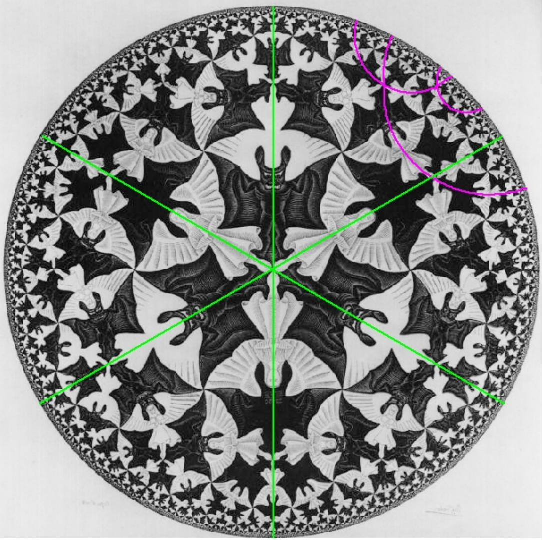
Write down as many definitions, ideas and concepts that relate to the meaning or visualization of parallel.



asapscience / Tumblr

Parallels in Various Geometries

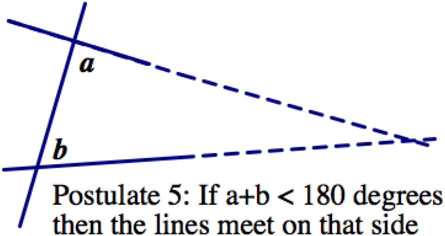
Escher's representation of hyperbolic geometry



Adapted from *Circle Limit 4: Heaven and Hell* by M.C. Escher, 1960

Euclid's Elements 5th Postulate

If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side...

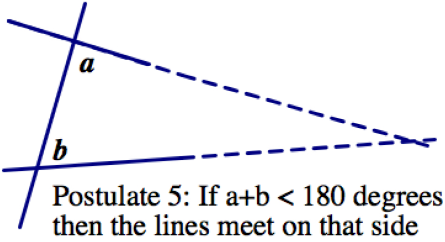


Postulate 5: If a+b < 180 degrees then the lines meet on that side

Guess the punchline!

Negation of Euclid's Elements 5th Postulate?

If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side...



Guess the punchline!

Consistency $\not\Rightarrow$ Uniqueness

Wile E. Coyote

1		1

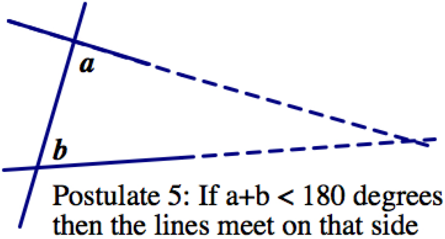
Axiom 1) Each square is a number or a mine.

Axiom 2) A numbered square represents the number of neighboring mines in the blocks immediately above, below, left, right, or diagonally touching (or a subset of those if a block is on a boundary)

How many consistent games can you find that satisfy the initial board plus the axioms?

Euclid's 5th Postulate on Plane and Sphere

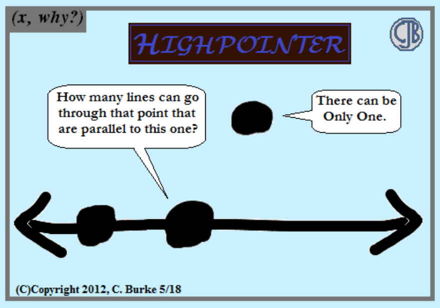
If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side...

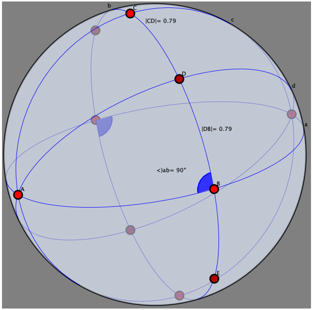


Guess the punchline!

Playfair's on Plane and Sphere

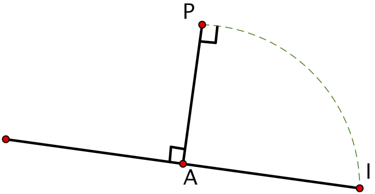
Given a line and a point off that line there is exactly 1 parallel to the line through the point.





Existence Portion of Playfair's Postulate I-31

- Create a parallel—what Euclidean propositions?



Let m be a line and P a point off it. To construct a parallel, first construct the perpendicular to m through P by I-12.

Existence Portion of Playfair's Postulate I-31

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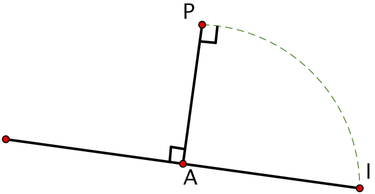

Let m be a line and P a point off it. To construct a parallel, first construct the perpendicular to m through P by I-12. Next apply I-11 to construct the perpendicular to the perpendicular through P.

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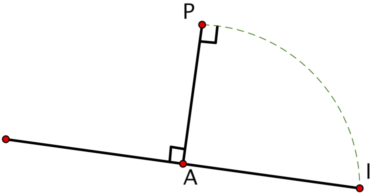

Let m be a line and P a point off it. To construct a parallel, first construct the perpendicular to m through P by I-12. Next apply I-11 to construct the perpendicular to the perpendicular through P. If, for contradiction, they intersect, then label the intersection as I and the other point of the resulting triangle A. We know $\sphericalangle API$ and $\sphericalangle PAI$ are right angles since we constructed them via perpendiculars. Now the exterior angle to the triangle across m at A must also be right by I-13. But

Existence Portion of Playfair's Postulate I-31

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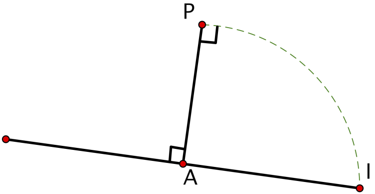

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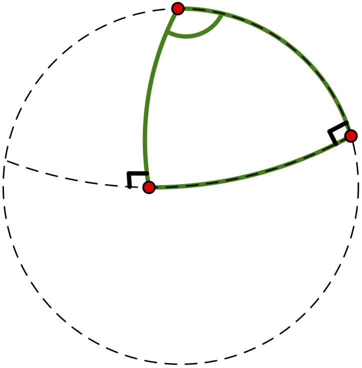
I-16 on the Sphere

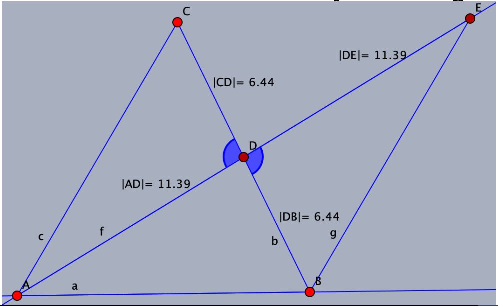
Why is the perpendicular to the perpendicular parallel in Euclidean geometry but not in spherical geometry?

I-16 An exterior angle of a triangle is greater than either remote interior angle.



Proof of I-16 on Plane & What Happens on Sphere

Let D be the midpoint of $\overline{BC}$ in triangle ABC, by I-10. Construct and extend $\overline{AD}$ by Postulates 1 & 2 and use I-2 to find E on it so $\overline{AD} = \overline{DE}$. Now $\sphericalangle ADC + \sphericalangle CDE$ is 2 right angles and so is $\sphericalangle BDE + \sphericalangle CDE$ by I-13 so the vertical angles are equal by CN3. We have SAS for triangles ACD and DBE so by I-4 $\sphericalangle ACD \cong \sphericalangle DBE$. In addition, the exterior angle at B strictly contains $\sphericalangle DBE$ so by CN5 is greater than it.



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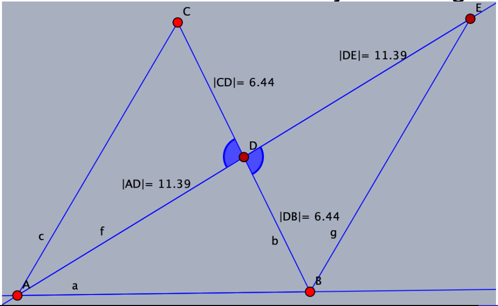
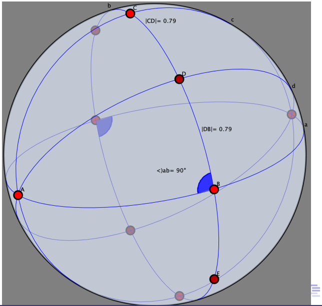
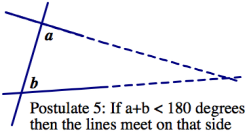




Existence Portion of Playfair's I-31

- Why is the perpendicular to the perpendicular parallel in Euclidean geometry but not in spherical geometry?
- Euclid's 5th Postulate is vacuously true on the sphere so unlike what is listed on the web and in some books, the statements are different. We will prove: Euclid's 5th Postulate plus Euclid's other axioms before I-31 prove Playfair's (underlying assumptions like for SAS!)
- We will further investigate parallels in hyperbolic geometry.



Guess the punchline!