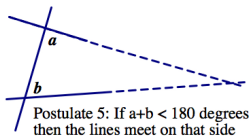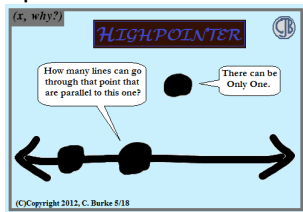# Parallel Axiom or Proposition?

Playfair's: Given a line and a point off that line there is exactly 1 parallel to the line through the point.

Euclid's 5th: If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side...

Prove that Playfair's plus Euclid's propositions before I-28, and prior content except Euclid's 5th, implies Euclid's 5th.
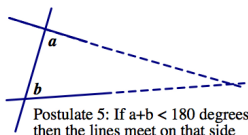
a

b

Postulate 5: If a+b < 180 degrees then the lines meet on that side

Guess the punchline!

(x, why?)

HIGHPOINTER

How many lines can go through that point that are parallel to this one?

There can be Only One.

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Proving Playfair's Postulate (I-31 + uniqueness)

Prove that Euclid's 5th Postulate plus Euclid's propositions before I-17, and prior content, prove Playfair's.
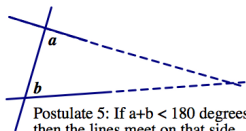
- Existence portion of Playfair's postulate (I-31)
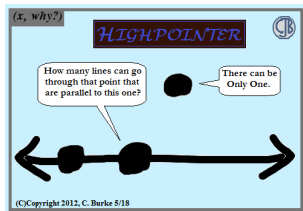- Show there can't be any additional parallels

Postulate 5: If a+b < 180 degrees then the lines meet on that side

Guess the punchline!

Dr. Sarah
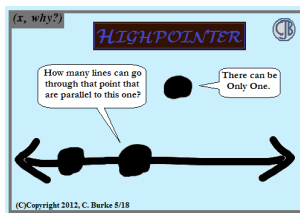
Why aren't they logically equivalent statements?

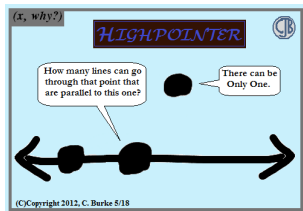Postulate 5: If a+b < 180 degrees then the lines meet on that side

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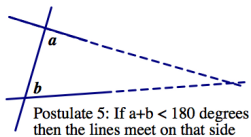
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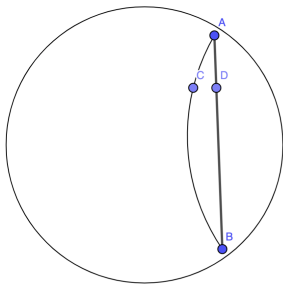
underlying assumptions—consider the sphere!

Hyperbolic Shortest Distance Paths?

Which were shorter? What happened to the difference between the distances? Can you obtain the different types of paths that Escher represented as cutting angels and demons in half?

hyperbolic distance from A to B through C=6.57557

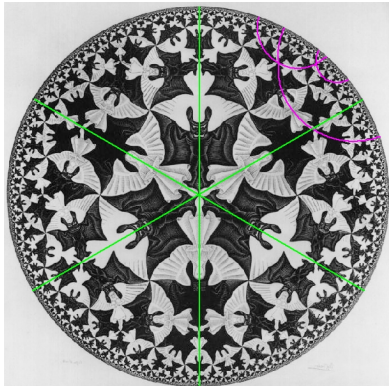hyperbolic distance from A to B through D=6.8381

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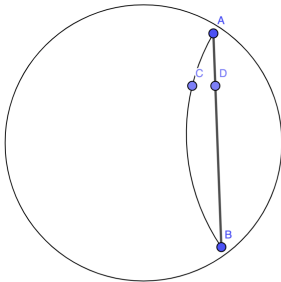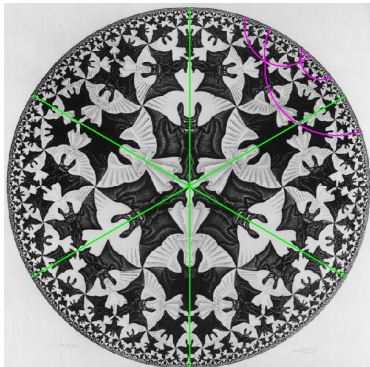hyperbolic distance from A to B through D=6.8381

Hyperbolic Angle Sum?
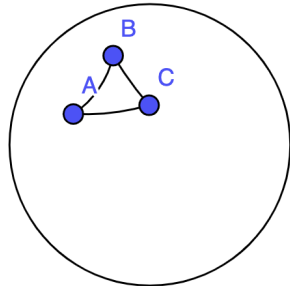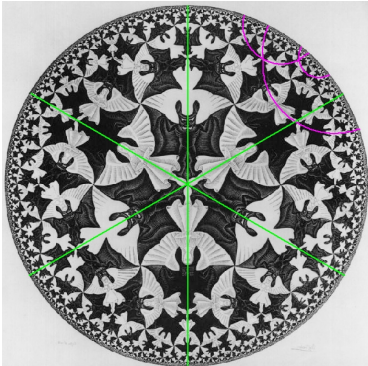
How did we compute the sum? How large can they get and what do the triangles look like? How small?

Hyperbolic Angle Sum?

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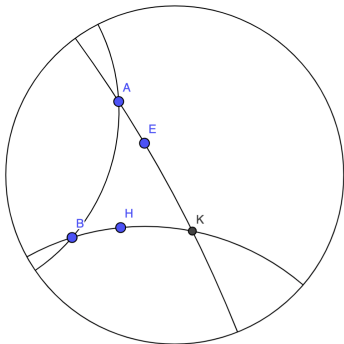what do the triangles look like? How small?

sum of the angles=144.99016

Hyperbolic Euclid's 5th?

Did Euclid's 5th always, sometimes, or never hold? What happens as we dragged *E*?

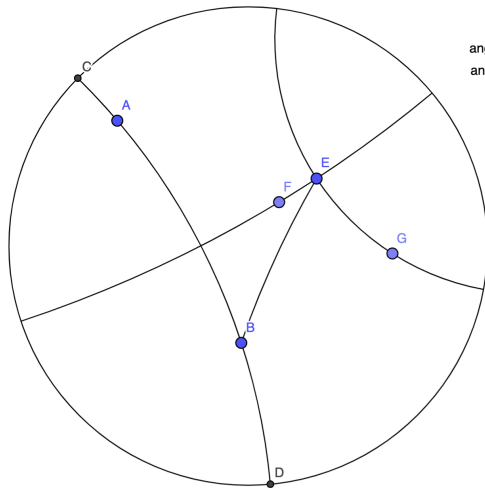measure angle EAB=29.78072

measure angle ABH=31.73487

Hyperbolic Playfair's?

Did the existence part of Playfair's hold? What happens to the alternate interior angles when cut by a transversal?

Hyperbolic Playfair's?

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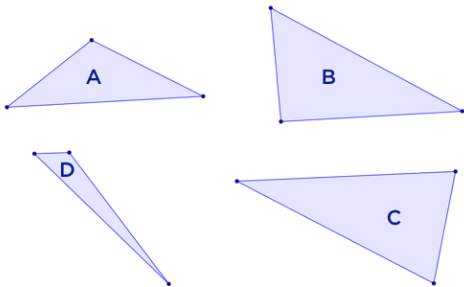

angle GEB = 63.31903
angle EBA=37.10285

SMSG 16 and Hyperbolic Angle Sum

- What happens in various geometries to SMSG Postulate 16: Through a given external point there is at most one line parallel to a given line.
- What goes wrong in the Euclidean proof of I-32 for the 180° angle sum in hyperbolic geometry?

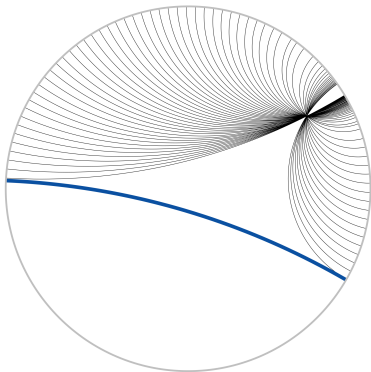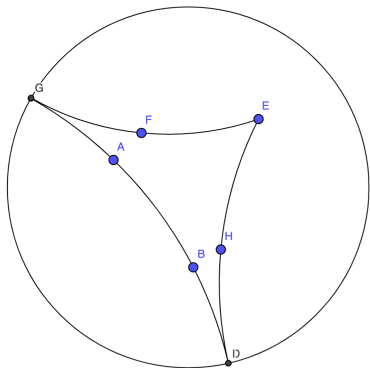**Which of these triangles has the greatest interior angle sum?**

Hyperbolic Parallel Axiom

If *l* is a hyperbolic line and *P* is a point not on *l*, then there exist exactly two noncollinear hyperbolic halflines which do not intersect *l* and such that a third hyperbolic halfline intersects *l* if and only if it is between the other two (and doesn't intersect it otherwise).
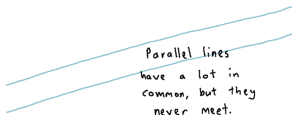
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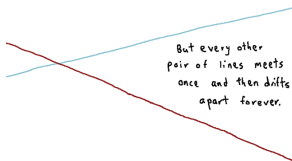
	Euclidean	hyperbolic	spherical
Euclid's 5th			
I-31			
Playfair's			
SMSG 16			

Parallel lines have a lot in common, but they never meet.

Ever.

You might think that's sad.

But every other pair of lines meets once and then drifts apart forever.

Which is pretty sad too.