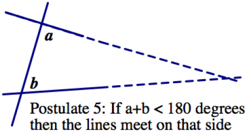
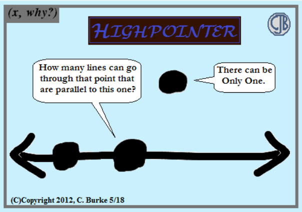
Parallel Axiom or Proposition?

Playfair's: Given a line and a point off that line there is exactly 1 parallel to the line through the point.
Euclid's 5th: If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side...

Prove that Playfair's plus Euclid's propositions before I-28 and prior content except Euclid's 5th implies Euclid's 5th.



Guess the punchline!



Playfair's + prior to I-28 $\rightarrow$ Euclid's 5th: Part 1

Assume Playfair's plus Euclid before I-28 (except Euclid's 5th). To show Euclid's 5th holds, assume $\overline{BE}$ is transversal to $\overline{AC}$ and $\overline{DF}$ at B & E with $\sphericalangle ABE + \sphericalangle BED < 2$ right angles. We must show that $\overline{AC}$ and $\overline{DF}$ meet on the A/D side.

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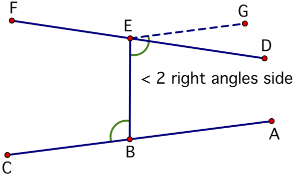

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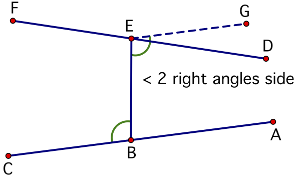
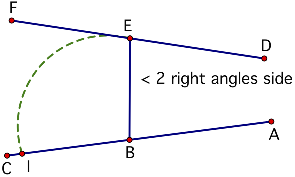


Examine $\sphericalangle GEB + \sphericalangle ABE \cong \sphericalangle CBE + \sphericalangle ABE \cong$ 2 right angles by I-13 and CN2. Thus $\overline{DF}$ is a different line through E than $\overline{EG}$ so $\overline{DF}$ must intersect $\overline{AC}$ because Playfair's says there is only 1 parallel to $\overline{AC}$ though E and $\overline{EG}$ is that parallel.

Playfair's + prior to I-28 $\rightarrow$ Euclid's 5th: Part 2

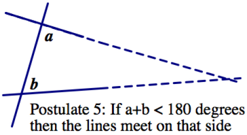


If, for contradiction, $\overline{DF}$ meets $\overline{AC}$ on the C/F side, then look the intersection point I. Apply I-17 to triangle IBE to conclude that $\sphericalangle IBE + \sphericalangle BEF < 2$ right angles. Then $\sphericalangle IBE + \sphericalangle BEF + \sphericalangle ABE + \sphericalangle BED < 2$ right angles + 2 right angles = 4 right angles by CNs. However, these 4 angles are also 2 sets of supplementary angles, so reorganizing them by CNs, we see that $\sphericalangle IBE + \sphericalangle ABE + \sphericalangle BEF + \sphericalangle BED = 2$ right angles + 2 right angles = 4 right angles by I-13. They can't be less than and equal to 4 right angles, so we have a contradiction and the intersection must be on the A/D side!

Proving Playfair's (I-31 + uniqueness)

Prove that Euclid's 5th Postulate plus Euclid's propositions before I-17, and prior content, prove Playfair's.

- Existence portion of Playfair's postulate (I-31)
- Show there can't be any additional parallels



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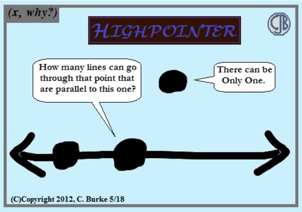
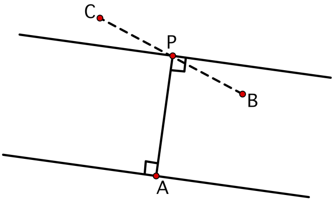


Euclid's 5th $\rightarrow$ uniqueness for Playfair's



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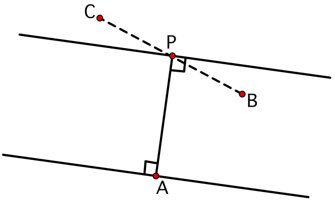

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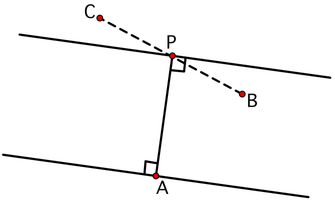

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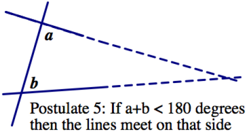
Why aren't they logically equivalent statements?



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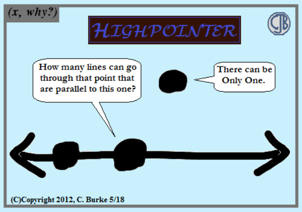

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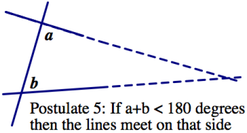

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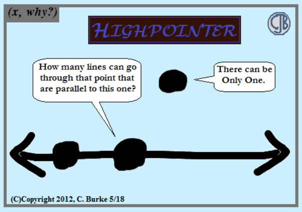


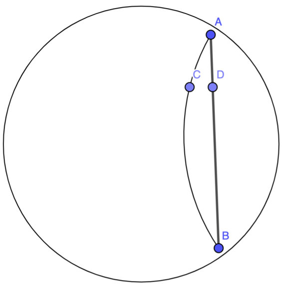
underlying assumptions—consider the sphere!

Hyperbolic Shortest Distance Paths?

Which were shorter? What happened to the difference between the distances? Can you obtain the different types of paths that Escher represented as cutting angels and demons in half?

hyperbolic distance from A to B through C=6.57557

hyperbolic distance from A to B through D=6.8381



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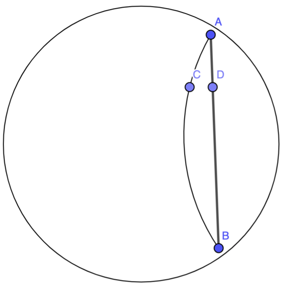
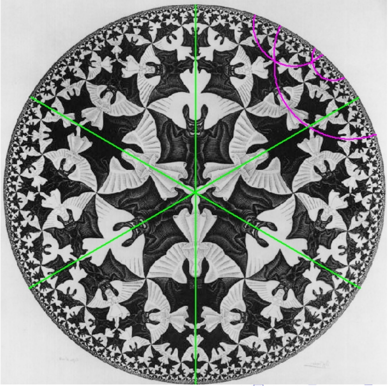
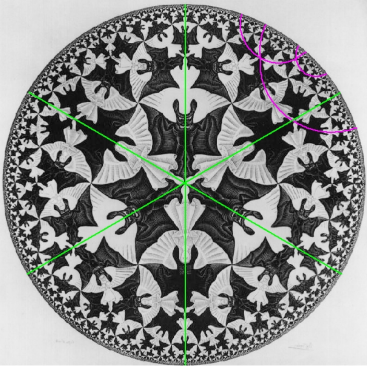




Hyperbolic Angle Sum?

How did we compute the sum? How large can they get and what do the triangles look like? How small?



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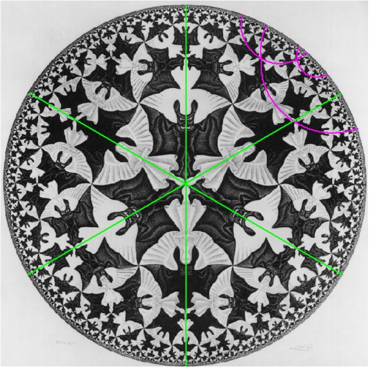
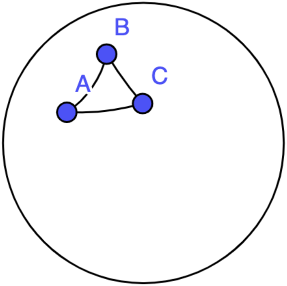


sum of the angles=144.99016

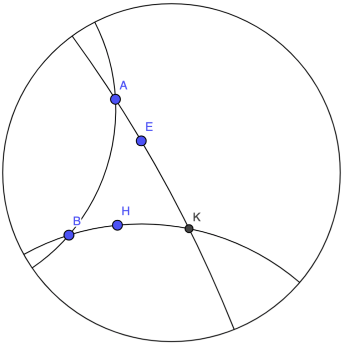


Hyperbolic Euclid's 5th?

Did Euclid's 5th hold in hyperbolic geometry? What happens as we dragged *E*?

measure angle EAB=29.78072

measure angle ABH=31.73487



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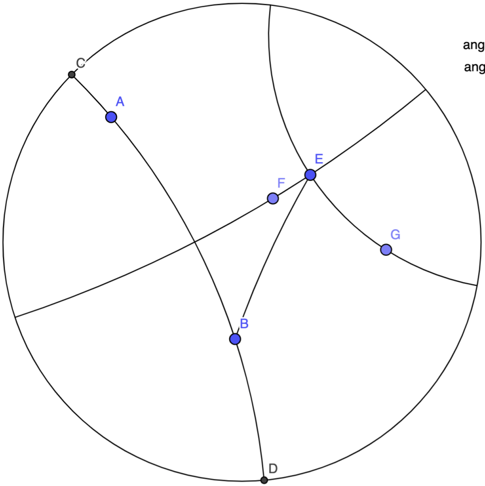
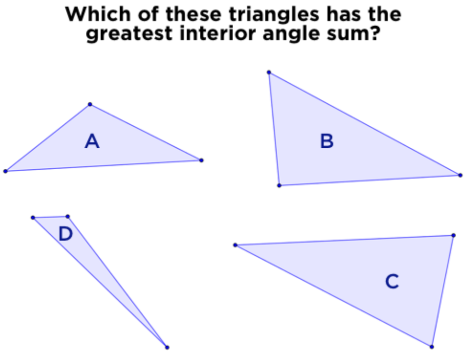


angle GEB = 63.31903

angle EBA=37.10285

SMSG 16 and Hyperbolic Angle Sum

- What happens in various geometries to SMSG Postulate 16: Through a given external point there is at most one line parallel to a given line.
- What goes wrong in the Euclidean proof of I-32 for the 180° angle sum in hyperbolic geometry?



https://blog.mrmeyer.com/2016/recipes-for-surprising-mathematics/

Hyperbolic Parallel Axiom

If l is a hyperbolic line and P is a point not on l, then there exist exactly two noncollinear hyperbolic halflines which do not intersect l and such that a third hyperbolic halfline intersects l if and only if it is between the other two (and doesn't intersect it otherwise).

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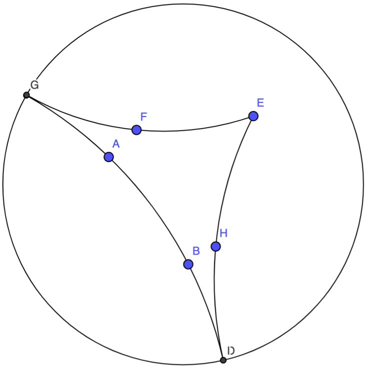
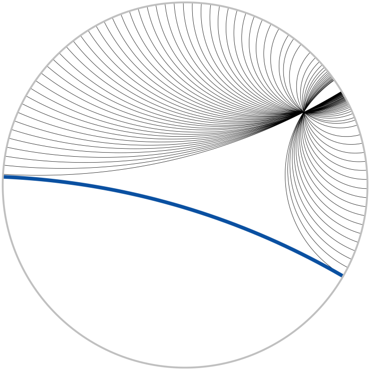
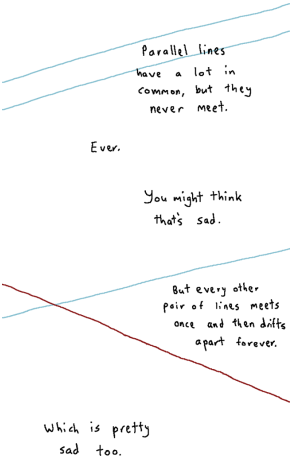




	Euclidean	hyperbolic	spherical
Euclid's 5th			
I-31			
Playfair's			
SMSG 16			



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