Equidistant

- Prove that parallel lines imply that they are equidistant in Euclidean geometry.
- What goes wrong on the sphere and in hyperbolic geometry?



Guess the punchline!



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Parallel Lines are Equidistant Let *a* and *b* be parallel lines. To show they are equidistant, let *A* and A' be points on a. Construct the perpendiculars to b through A and A' by

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Parallel Lines are Equidistant Let *a* and *b* be parallel lines. To show they are equidistant, let *A* and A' be points on a. Construct the perpendiculars to b through A and A' by I-12 and label the intersections as B and Β'.

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In addition, $\triangleleft ABB' \cong \triangleleft AA'B$ since they are right angles by Postulate 4 and $\overline{AB'} = \overline{AB'}$ by CN4. Hence the triangles are congruent by

Parallel Lines are Equidistant Let *a* and *b* be parallel lines. To show they are equidistant, let *A* and A' be points on a. Construct the perpendiculars to b through A and A' by I-12 and label the intersections as B and B'. Construct $\overline{AB'}$ by Postulate 1. Now $\triangleleft A'AB' \cong \triangleleft AB'B$ by I-29.



In addition, $\triangleleft ABB' \cong \triangleleft AA'B$ since they are right angles by Postulate 4 and $\overline{AB'} = \overline{AB'}$ by CN4. Hence the triangles are congruent by I-26 (AAS) and so $\overline{AB} \cong \overline{A'B'}$.

- What first goes wrong in hyperbolic geometry?
- What goes wrong in spherical geometry?

What are the shortest distance paths in Escher's model of hyperbolic geometry?

- a) perpendicular to the boundary at infinity
- b) symmetric paths that cut through the center of creatures
- c) feel straight intrinsically
- d) more than one of the above but not all of the above
- e) all of a), b) and c)

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In the existence portion of Playfair's, to show that a parallel exists, we used:

- a) perpendicular to the perpendicular to the line
- b) Euclid's Elements I-11 and I-12
- c) Euclid's Elements I-16
- d) more than one of the above but not all of the above
- e) all of a), b) and c)

What goes wrong, if anything, with the existence portion of Playfair's in spherical geometry?

- a) Euclid's Elements I-11
- b) Euclid's Elements I-12
- c) Euclid's Elements I-16
- d) more than one of the above but not all of the above
- e) all of a), b), c) or none of the above

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https://www.geogebra.org/m/xynfrc93

Pythagorean in Hyperbolic Geometry



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Pythagorean in Hyperbolic Geometry





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Pythagorean in Hyperbolic Geometry



Representing Geometries



Dutch graphic artist M.C. Escher's Sphere Surface with Fish, 1958 and Circle Limit IV: Heaven and Hell, 1960; Latvian/US mathematician Daina Taimina *Crocheting Adventures with Hyperbolic Planes*

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Does the real universe have curves? Euclidean?



Mike Peters https://www.grimmy.com/comics.php?sel_dt=2012-05-21

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Scientific & Mathematical Breakthroughs

- They require imaginative leaps
- Understanding what we are seeing is complicated by filters

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Angle Sum Experiments

• Carl Frederich Gauss (1777–1855): Hoher Hagen, Inselsberg, and Brocken $180^{\circ} \pm \frac{1}{180}$

Ranelsen - Hoher Hagen - Inselstern Das größte von Carl Friedrich Gauß vermessene Dreieck im Zuge der hannoverschen Gradmessung (1821 - 1825)zur Bestimmung der Erdgestalt.

• Nikolai Lobachevsky (1792–1856): star Sirius 180° – sum of the angles = 3.727×10^{-6} (should be 10^{-8})

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- Nikolai Lobachevsky (1792–1856): star Sirius 180° sum of the angles = 3.727×10^{-6} (should be 10^{-8}) Euclidean=180°, spherical> 180°, hyperbolic< 180°
- Critiques: margin of error, light rays bend with gravity, triangles too small, convenience sample





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Distant supernovae dimmer than expected in Euclidean

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Distant supernovae dimmer than expected in Euclidean Critiques: Experimental error, no perfect model, not necessarily exploding at the same brightness

Density Experiments: WMAP & Planck

- Cosmic Microwave Background: small temperature fluctuations due to primordial plasma density
- Density equation
- Infinite Euclidean universe 0 \pm .4%

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Density Experiments: WMAP & Planck

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- Critiques: convenience samples, observable, experimental error, difficulty agreeing on the meaning of the data, neutrino mass, dark energy, speed of light?

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Wilkinson Microwave Anisotropy Probe (WMAP)



Spherical Dodecahedron?

historically Platonic solids: universe = finite dodecahedron



Paul Nylander: life from the inside $\langle \Box \rangle + \langle \Box \rangle + \langle \Box \rangle + \langle \Xi \rangle + \langle \Xi \rangle = \langle \neg \rangle < \bigcirc$

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Hyperbolic Structures?



Jos Leys

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Applications of Hyperbolic Geometry

Models of the internet to reduce the load on routers



Sustaining the Internet with hyperbolic mapping: Marian Boguna, Fragkiskos Papadopoulos & Dmitri Krioukov

Building crystal structures to store more hydrogen or absorb more toxic metals

Modeling and Explaining Real-Life Behavior



www.sciencenews.org/article/einsteins-genius-changed-sciences-perception-gravity

- hyperbolic geometry better models Mercury's orbit
- both Euclidean and non-Euclidean geometry map the brain to diagnose or monitor neurological diseases

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Modeling and Explaining Real-Life Behavior



life.dpics.org

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