## Equidistant

- Prove that parallel lines imply that they are equidistant in Euclidean geometry.
- What goes wrong on the sphere and in hyperbolic geometry?


Guess the punchline!


## Parallel Lines are Equidistant

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In addition, $\varangle A B B^{\prime} \cong \varangle A A^{\prime} B$ since they are right angles by Postulate 4 and $\overline{A B^{\prime}}=\overline{A B^{\prime}}$ by CN4. Hence the triangles are congruent by

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- What first goes wrong in hyperbolic geometry?
- What goes wrong in spherical geometry?

What are the shortest distance paths in Escher's model of hyperbolic geometry?
a) perpendicular to the boundary at infinity
b) symmetric paths that cut through the center of creatures
c) feel straight intrinsically
d) more than one of the above but not all of the above
e) all of a), b) and c)

In the existence portion of Playfair's, to show that a parallel exists, we used:
a) perpendicular to the perpendicular to the line
b) Euclid's Elements I-11 and I-12
c) Euclid's Elements I-16
d) more than one of the above but not all of the above
e) all of a), b) and c)

What goes wrong, if anything, with the existence portion of Playfair's in spherical geometry?
a) Euclid's Elements I-11
b) Euclid's Elements I-12
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https://www.geogebra.org/m/xynfrc93

## Pythagorean in Hyperbolic Geometry



## Pythagorean in Hyperbolic Geometry

## Poincaré Disk Model

This sketch depicts the hyperbolic plane $H^{2}$ using the Poincare disk model. In this model, a line through Uso this document's custom tools to perform constructions on the hyperbolic plane, comparing your findings to equivalent constructions on the Eucidean plane.

Can we construct a square? $\mathrm{m} \angle \mathrm{ABC}=90.0^{\circ}$
$\mathrm{m} \angle \mathrm{BCD}=90.0^{\circ}$
$A B=1.14$
$B C=1.14$
$C D=1.14$
$A D=1.81$
$m \angle C D A=54.2^{\circ}$


2006
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Dr. Sarah

## Pythagorean in Hyperbolic Geometry

## Poincaré Disk Model

This sketch depicts the hyperbolic plane $H^{2}$ using the Poincare disk model. In this model, a line through two points is defined as the Euclidean arc passing through the points and perpendicular to the circle. to equivalent constructions on the Euclidean plane.

## Disk Controls



Dr. Sarah

## Representing Geometries



Dutch graphic artist M.C. Escher's Sphere Surface with Fish, 1958 and Circle Limit IV: Heaven and Hell, 1960;
Latvian/US mathematician Daina Taimina Crocheting Adventures with Hyperbolic Planes

## Does the real universe have curves? Euclidean?



Mike Peters https://www.grimmy.com/comics.php?sel_dt=2012-05-21

## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Venus



## Scientific \& Mathematical Breakthroughs

- They require imaginative leaps
- Understanding what we are seeing is complicated by filters

Nicolaus Copernicus (1473-1543): Heliocentric Model


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## Angle Sum Experiments

- Carl Frederich Gauss (1777-1855): Hoher Hagen, Inselsberg, and Brocken $180^{\circ} \pm \frac{1}{180}$

- Nikolai Lobachevsky (1792-1856): star Sirius $180^{\circ}$ - sum of the angles $=3.727 \times 10^{-6}\left(\right.$ should be $\left.10^{-8}\right)$


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- Nikolai Lobachevsky (1792-1856): star Sirius $180^{\circ}$ - sum of the angles $=3.727 \times 10^{-6}$ (should be $10^{-8}$ ) Euclidean $=180^{\circ}$, spherical $>180^{\circ}$, hyperbolic $<180^{\circ}$
- Critiques: margin of error, light rays bend with gravity, triangles too small, convenience sample


## Supernova Experiments



Euclidean inverse square law: brightness $\sim \frac{1}{\text { distance }^{2}}$


Euclidean inverse square law: brightness $\sim \frac{1}{\text { distance }^{2}}$ Hyperbolic < and spherical >


Distant supernovae dimmer than expected in Euclidean


Euclidean inverse square law: brightness $\sim \frac{1}{\text { distance }^{2}}$ Hyperbolic < and spherical >

Distant supernovae dimmer than expected in Euclidean Critiques: Experimental error, no perfect model, not necessarily exploding at the same brightness

## Density Experiments: WMAP \& Planck

- Cosmic Microwave Background: small temperature fluctuations due to primordial plasma density
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- Density equation
- Infinite Euclidean universe $0 \pm .4 \%$
- Missing fluctuations on large scale better fit a large spherical dodecahedral space [Jeff Weeks] or hyperbolic [Ron Cowen]
- Critiques: convenience samples, observable, experimental error, difficulty agreeing on the meaning of the data, neutrino mass, dark energy, speed of light?


## Wilkinson Microwave Anisotropy Probe (WMAP)



## Spherical Dodecahedron?

historically Platonic solids: universe = finite dodecahedron


Paul Nylander: life from the inside

## Hyperbolic Structures?



Jos Leys

## Applications of Hyperbolic Geometry

- Models of the internet to reduce the load on routers


Sustaining the Internet with hyperbolic mapping: Marian Boguna, Fragkiskos Papadopoulos \& Dmitri Krioukov

- Building crystal structures to store more hydrogen or absorb more toxic metals


## Modeling and Explaining Real-Life Behavior


www.sciencenews.org/article/einsteins-genius-changed-sciences-perception-gravity

- hyperbolic geometry better models Mercury's orbit
- both Euclidean and non-Euclidean geometry map the brain to diagnose or monitor neurological diseases


## Modeling and Explaining Real-Life Behavior


life.dpics.org


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