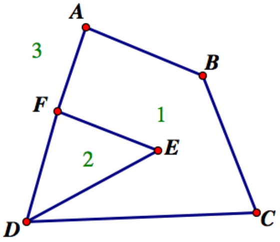
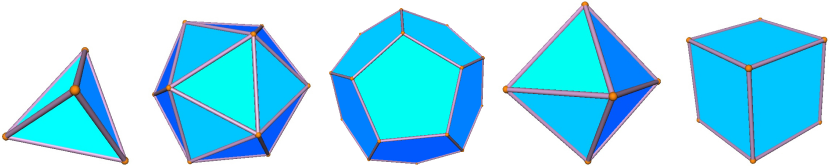
# V-E+F Experiment

- Draw a few dots (vertices)
- Connect the dots with lines, subject to the following rules:
  - lines may not cross each other as they move from dot to dot
  - every dot must be connected to every other dot through a sequence of lines
  - every region must topologically be a disk with no holes
- Compute
  Vertices (V) - Edges (E) + Faces Separated by Edges (F)
  [Do not forget to count the outside as a region for F too.]



# *Regular Polyhedra*

- What is Vertices (V) - Edges (E) + Faces (F) for the regular polyhedra?
- Where is the symmetry of a $\frac{2\pi}{3}$ rotation for each polyhedra? Describe the axis of rotation in each case.



Tetrahedron Icosahedron Dodecahedron Octahedron Cube

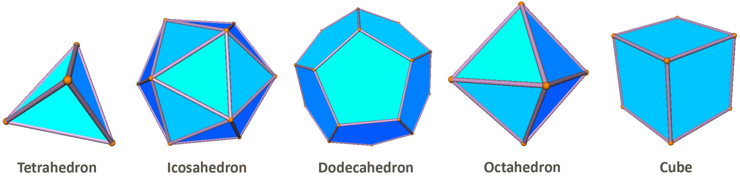
http://www.princeton.edu/pr/pictures/l-r/packingproblem/pu-platonic-solids.jpg

# Regular Polyhedra

One may think at first that one can construct an infinite number of regular polyhedra in three-dimensions, just as we could construct an infinite number of regular polygons in two-dimensions. However, this does not turn out to be the case.

There are only five regular polyhedra, but why?

*So their combinations with themselves and with each other give rise to endless complexities, which anyone who is to give a likely account of reality must survey.* [Plato, The Timaeus]

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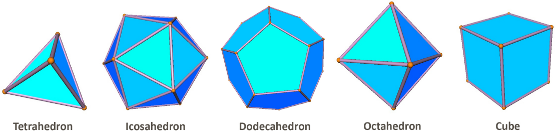


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# Spherical Icosahedron



- William O. Gustafson / Uwe Meffert
- 23 563 902 142 421 896 679 424 000 combinations
- V=12, E=30, F=20

# *Infinitely Many Regular Spherical Polyhedra*

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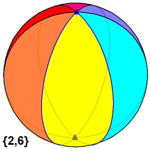


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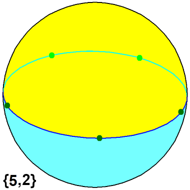




Pentagonal dihedron