Project 3: Similarity and Connections

The purpose of this problem set is to make connections and apply the material. You may work alone or in a group of 2 people. Groups turn in one project writeup. Be sure to show work and explain your reasoning in your own words. In addition, be sure to acknowledge any sources outside me or your group, like "The insight for this solution came from a conversation with Joel."

1. SMSG Postulates and *Euclid's Elements* (Geometric Perspectives)

   (a) Take a look at the very back of the book at the SMSG Postulates for Euclidean Geometry. Which postulates correspond to postulates 1–5 in Book I of *Euclid's Elements* (the handout).

   (b) Compare and contrast their statements.

   (c) Are any of *Euclid's Elements* postulates not represented here?

   (d) Which of the SMSG postulates relate to congruence or similarity of regions?

   (e) Research the history of the SMSG postulates for Euclidean Geometry and summarize what you find.

2. Midpoints of Quadrilateral (Interactive Geometry Software and Proof Considerations)

   (a) 4.2 #23 on p. 136 should look familiar from the first day of class. Create that figure in an IGS, measure the angles of the proposed parallelogram, and drag to show it seems to approximately be a parallelogram for a wide variety of examples. Select one view and print your sketch.

   (b) What is the definition of a parallelogram on p. 130?

   (c) To prove 4.2 #23, start your proof with the figure as given, i.e. let $ABCD$ be a quadrilateral and $EFGH$ be a figure with E as the midpoint of $\overline{CD}$, F as the midpoint of $\overline{BC}$, G as the midpoint of $\overline{AB}$, and H as the midpoint of $\overline{AD}$. Then connect $\overline{BD}$ (which postulate?) and prove that $\overline{GH}$ is parallel to $\overline{BD}$—make use of a congruence or similarity theorem from class and show you have satisfied the assumptions [do not assume Theorem 4.2.15 holds, i.e. ignore the book's hint]. Next show that $\overline{EF}$ is parallel to $\overline{BD}$. Continue similarly and write a complete proof that $EFGH$ is a parallelogram in paragraph form, including identifying any underlying assumptions. You may freely use items from

   - the handout of *Euclid's Elements* Book 1
   - similarity postulates and definitions from class

   (d) Read through Theorem 4.2.4 on p. 130 and answer the question the book posed in the second line by giving the number of a proposition in the handout of *Euclid's Elements* Book 1.

   (e) What numbered congruence proposition from the handout of *Euclid's Elements* Book 1 did the book use in the fourth line of the proof of Theorem 4.2.4?

3. Euclidean Quadrilateral Centroids (Interactive Geometry Software)

   (a) Construct a circle with center A and radius AB, i.e. B is a point on the circle.

   (b) Construct 3 additional points on the circle, C, D, E so that C is somewhere in between B and D, and D is in between C and E.

(c) Connect the edges to form a quadrilateral $BCDE$. ***and make sure none of the angles are right angles.

(d) Construct the four centroids of the four various triangles formed by the diagonals. A centroid of a triangle is the intersection of the three lines from each vertex to the midpoint of the opposite side. You can use the following tools:

- Midpoint or Center
- Segment
- Intersect
- Show/Hide Object

(This is similar to what we did for orthocenters in lab, but create the centroids instead.)

(e) Construct the quadrilateral formed by the centroids. Drag some points to see whether the relationships seem to hold in a wide variety of examples.

(f) What do you notice in terms of congruence or similarity or neither? Explain.

(g) Measure using the Distance or Length tool to see whether you can find any relationships between the sides of the centroid quadrilateral and the original quadrilateral.

(h) Can you find an (approximate) dilation ratio among the quadrilaterals? Explain.

(i) Select one view to print.

4. Logger Measurements (Geometric Perspectives)

Excel Regression Reminders and Data

- Enter in the data so that the x-axis is in a column before the y-axis
- Formulas in Excel are entered as (for example) =b2∧(2/3)
- To insert a new column, click on the next column and Insert/Column
- To fill a formula down a column, click on the first box, go to bottom right, click when the symbol turns to a black plus sign, and fill down.
- To create a linear regression plot, click on the letters above the columns, then Insert/Chart/scatter/.
- Control click on one of the points on the Chart, choose Add Trendline, and then choose Display R-square value and hit OK.

diameter of a ponderosa pine in inches	board feet/10 in cubic inches
17	19
19	25
20	32
23	57
25	71
28	113
32	123
38	252
39	259
41	294

(a) Assume all trees are right circular cylinders and all trees are about the same height. Let r be the radius, h the height and k a constant of proportionality:

volume $= \pi r^2 h = \pi(\frac{\text{diameter}}{2})^2 h = \pi \frac{\text{diameter}^2}{4} h = k$ diameter$^2 \propto$ board feet/10
So test out the model in Excel by relating diameter^2 to board feet/10. Create a regression plot with an R^2 value.

(b) Assume instead that all trees are still right circular cylinders but that the height of the tree is now proportional to the diameter. Create a model relating diameter^x to board feet/10, i.e. specify x and show work.

(c) Test this second model in Excel. Create a regression plot with an R^2 value.

(d) Print your two graphs (they can all be on one page) with the R^2 values shown.

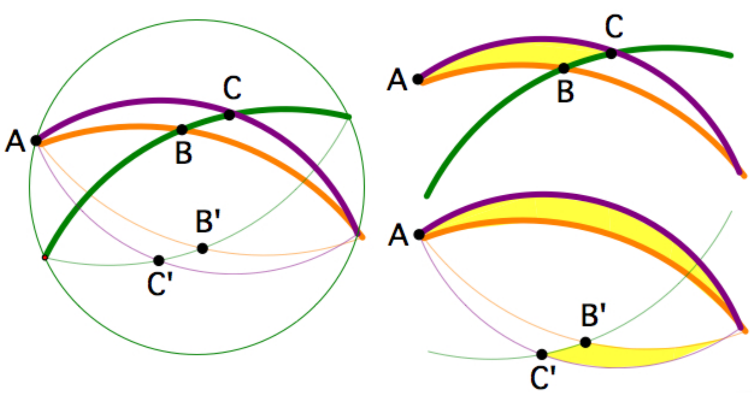
(e) Which model appears better in Excel and why? Explain.

(f) Regardless of the Excel work, which assumption appears the most reasonable in real-life? Why?

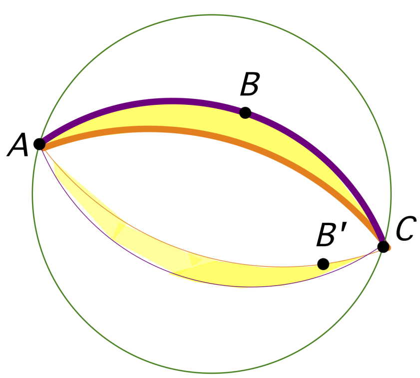
(g) What are some other underlying assumptions and factors that we could consider in real-life?

5. AAA (Geometric Perspectives and Proof Considerations)
Examine the Euclidean proof of AAA via Theorem 4.4.5 on p. 149–150 in the book [there is a typo as the second sentence should say ASA rather than SAS] and consider where this proof goes wrong on the sphere—we found two pairs of counterexamples of spherical triangles that satisfy the conditions of AAA but are not similar:

pair 1: a usual triangle and a self-intersecting triangle



pair 2: two triangles with angles all 180°



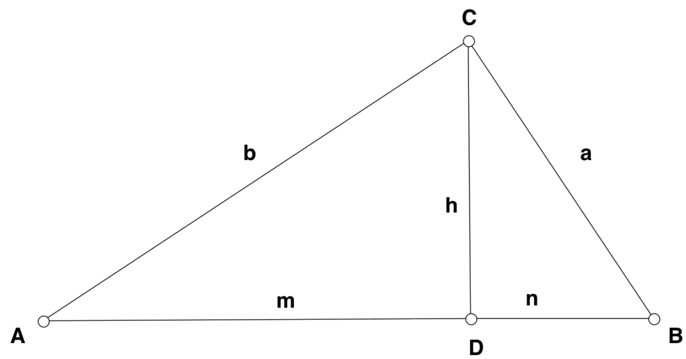
Select <u>one</u> of these pairs. Relate the counterexample to the Euclidean proof in the book to discuss what goes wrong with the Euclidean proof in the spherical pair [similar to what we did for the Euclidean SAS proof and two spherical triangles that satisfied the assumptions of SAS but not the proof or conclusions].

6. Bhāskarāchārya (Geometric Perspectives and Proof Considerations)

Bhāskarāchārya (1114–1185), an Indian mathematician and astronomer, devised a proof of the Pythagorean Theorem based upon the notion of similar triangles (also see p. 151-152).

See below and fill in the details and reasons. You may freely refer to items on

- the handout of *Euclid's Elements* Book 1
- similarity postulates and definitions from class



Consider a right triangle ABC with sides a, b, and c. (We will continue to assign the shorter leg as a and the hypotenuse c.) For ease in the construction, orient the triangle as shown in the diagram to the right.

Construct the perpendicular from point C to the hypotenuse at point D. Call the length of segment CD "h", the length of segment AD "m", and the length of segment DB "n".

Triangles ABC, CBD, and ACD are similar. (This is easy to show if you wish to do so.) Therefore, the ratio of hypotenuse to longer leg must be the same for each triangle; in particular, let's look at triangles ABC and ACD:

$$\frac{c}{b} = \frac{b}{m}$$

By cross-multiplying, we get Equation 1:

$$cm = b^2$$

Furthermore, the ratio of hypotenuse to shorter leg must also be the same for each triangle; in particular, let's look at triangles ABC and CBD:

$$\frac{c}{a} = \frac{a}{n}$$

By cross-multiplying, we get Equation 2:

$$cn = a^2$$

By adding equals to equals (Equation 1 + Equation 2), we have

$$cn + cm = a^2 + b^2$$
$$c(n + m) = a^2 + b^2$$
$$c^2 = a^2 + b^2$$

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