Project 5: Polyhedral and Analytic/Metric Geometry Considerations

The purpose of this problem set is to make connections and apply the material. You may work alone or in a group of 2 people. Groups turn in one project writeup. Be sure to show work and explain your reasoning in your own words. In addition, be sure to acknowledge any sources outside me or your group, like "The insight for this solution came from a conversation with Joel."

1. Look at the triangle whose vertices are $A = (2, 2), B = (-1, 1)$, and $C = (1, -1)$.
   (a) Show that this triangle is equilateral under the taxicab metric. Show work.
   (b) Is the triangle equilateral or isosceles under the Euclidean metric? Show work.

2. To show that the Pythagorean Theorem fails in taxicab geometry but still holds for some triangles:
   (a) Look at the triangle whose vertices are $A = (-3, 9), B = (12, 4)$ and $C = (0, 0)$. Notice that the slope of $\overline{AC}$ = -3 while the slope of $\overline{BC} = \frac{1}{3}$ and so $\overline{AC}$ is perpendicular to $\overline{BC}$. Hence this is a right triangle. Let c be the hypotenuse opposite vertex C, a be the side opposite vertex A and b be the side opposite vertex B. Compute a, b and c under the taxicab metric. Compare $a^2 + b^2$ with c^2. Does the Pythagorean theorem hold for this right triangle in taxicab geometry? Show work.
   (b) Look at the triangle whose vertices are $A = (0, 0), B = (4, 3)$ and $C = (4, 0)$. Notice that this is a right triangle. Compare $a^2 + b^2$ with c^2. Does the Pythagorean theorem hold for this right triangle in taxicab geometry? Show work.

3. A person in a city works at $W = (1, 0)$, eats regularly at $E = (8, 3)$, and does their laundry at $L = (7, 2)$.
   (a) If they want to rent a room R so as to be at the same walking distance from each of these points—using the taxicab metric—where could R be located? Show work.
   (b) Is there more than one possible answer for Part A? Explain.
   (c) Assume they rent a shortest distance room with the same walking distance to each of the points W, E, L. How many blocks do they have to walk from the room to them? Show work.
   (d) If all conceivable shortcuts are possible using the Euclidean metric, show work for the person's room to be located so that it is equidistant—use the intersection of the perpendicular bisectors and show work (hint: perpendiculars have negative reciprocal slopes and so you can write the equations of the perpendicular bisectors and intersect them).
   (e) What is the Euclidean measurement of the distance that they have to walk from the room in Part D to each of the three points? Show work.
   (f) Which metric gives a shorter walk?

4. (a) Without assuming that you already know what the 5 Euclidean Platonic solids are, use Euler's Formula and the equations for E and V in terms of n, k and p to prove that if a polyhedron has three triangular faces meeting at each vertex then it must have a total of four faces. Identify underlying assumptions.
   (b) Does your proof work on the sphere? Explain.

5. The Archimedean solids are solids whose faces are regular polygons (but not all the same) such that every vertex is symmetric with every other vertex.

   (a) Make a model (paper nets, or toothpicks and gumdrops, or any model you like!) of one of the Archimedean solids and bring it to class.

   (b) Roughly sketch the solid yourself in your writeup.

   (c) Research and list at least 2 different names of the solid you made and describe how you made it (you will present the model in class but I won't keep it).

   (d) List the types of polygon faces.

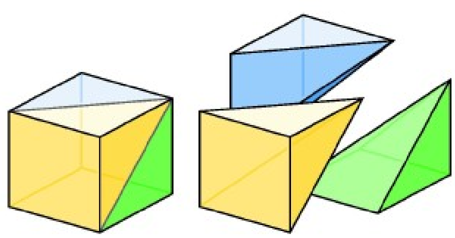
   (e) How many of each kind of face are there?

   (f) Specify the total number of vertices, edges, and faces and show that $V - E + F = 2$.

   (g) Discuss two different symmetries of your Archimedean solid and sketch accompanying pictures that you create yourself. As Susan Goldstine explains in "What the Origami Means,"

      > A symmetry of a polyhedron is a way of moving the polyhedron so that it occupies the same physical space as before it was moved. The symmetries of a polyhedron reflect its structure and regularity.

6. (a) This shows a cube decomposed along the diagonal into 3 square pyramids (the bottom square of the cube is the base of one, the square face towards us is another base, and the square face on the left side is the third). This could explain why the volume of a square pyramid is one-third the height times the base.

   To see whether they are congruent pyramids, use this pyramid net at `http://cs.appstate.edu/~sjg/class/3610/projects/three_pyramids_cube.gif`. Make 3 copies. Do these congruent pyramids approximately fit together along the diagonal of the cube? You can also shape material like clay or Play-Doh.

   (b) Can you find a decomposition of a rectangular box along the diagonal into three pyramids? Explain why or why not and what you tried. If you can find such a decomposition, are the three pyramids the same shape or different shapes? Explain and/or show sketches.

7. (Individual component) Research one item related to polyhedra and/or analytic geometry/metric geometry and focuses in your major(s) or intended careers and report back on it. Include the source.