Division

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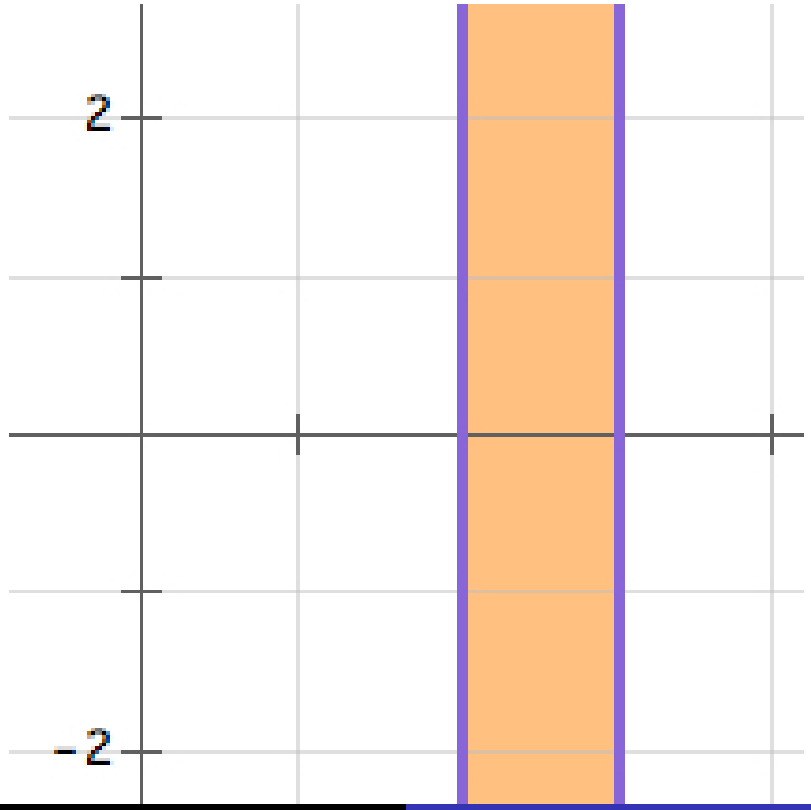
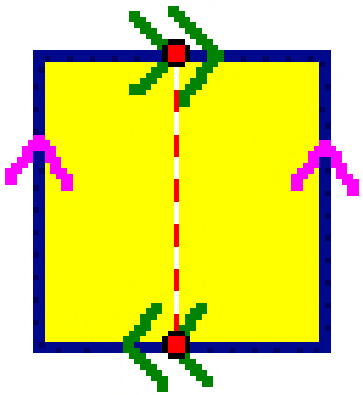
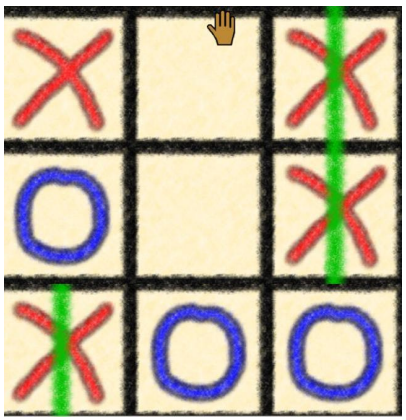
- Pizza - from a cut, $\frac{2\pi}{8} = 45$ degrees
- Plane: $(x, y) \to (x + 1, y)$



Surfaces that Locally Look Like the Plane?

- Felix Klein posed the question in 1890
- In Klein's Erlangen Program, the properties of a space were understood by the transformations that preserved them.
- Heinz Hopf's rigorous solution was 1925
  A complete connected surface which locally looks like the plane is obtained via a quotient by a group of isometries acting without fixed points

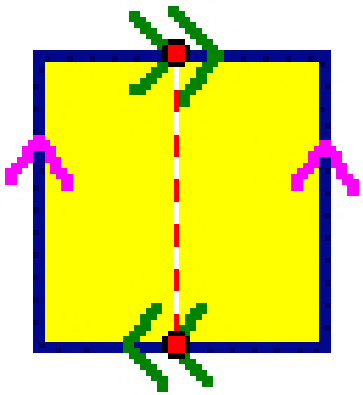


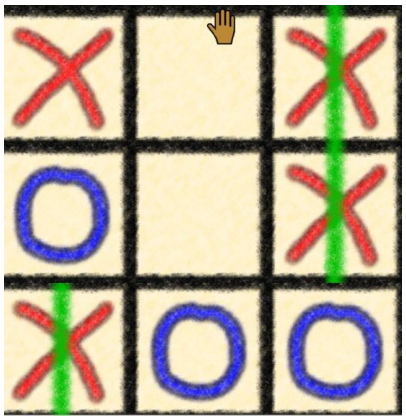


Surfaces that Locally Look Like the Plane

- Heinz Hopf's rigorous solution was 1925
  A complete connected surface which locally looks like the plane is obtained via a quotient by a group of isometries acting without fixed points

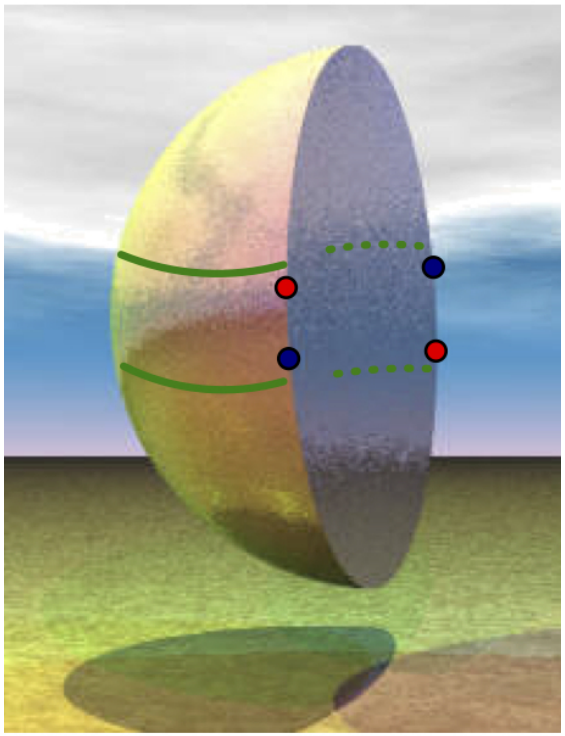
  Two points are the same if and only if we can get from one point to the other by a transformation: plane, cylinder, Mobius Band, flat Clifford torus, flat Klein bottle

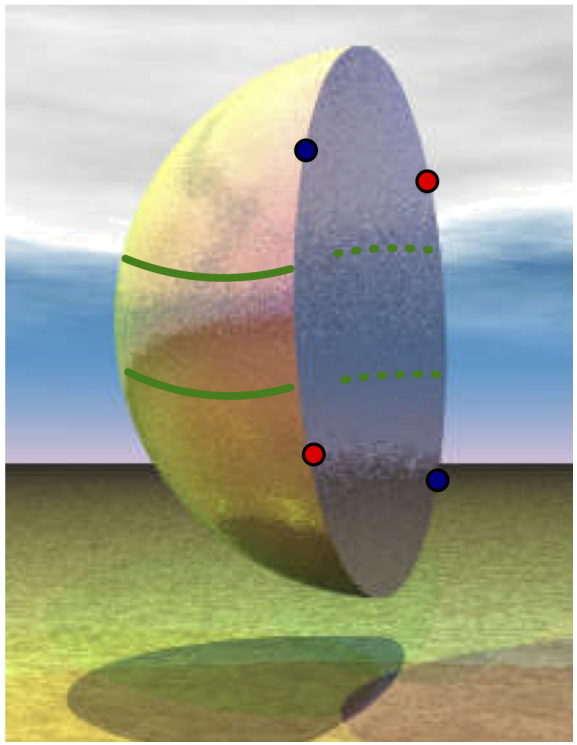




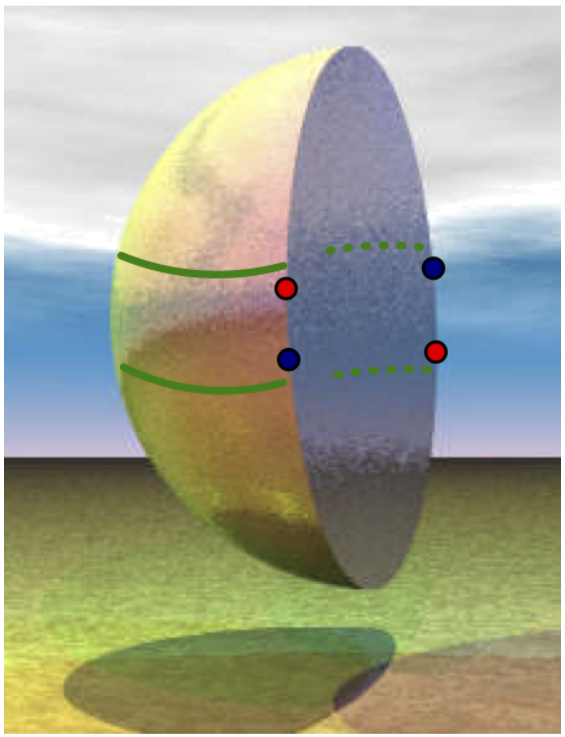
Surfaces that Locally Look Like the Sphere: $\mathbb{R}P^2$

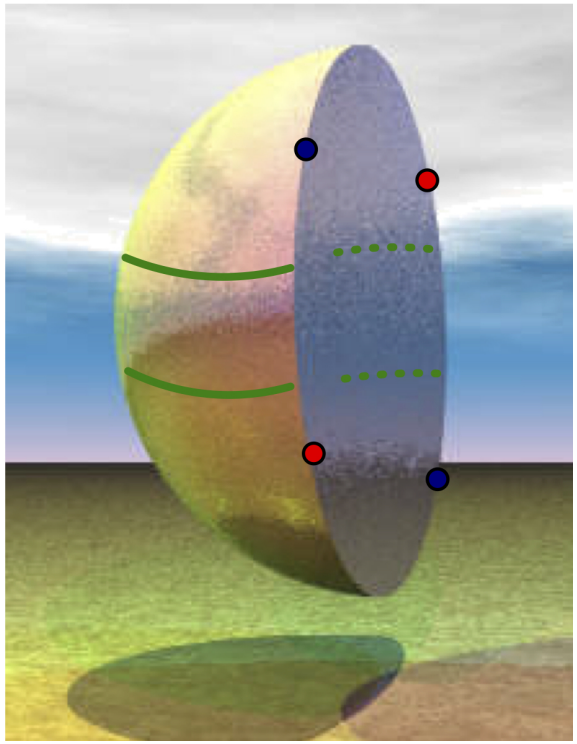
- $\frac{S^2}{\Gamma}$ where $\Gamma = \{\text{identity}, (x, y, z) \to (-x, -y, -z)\}$.





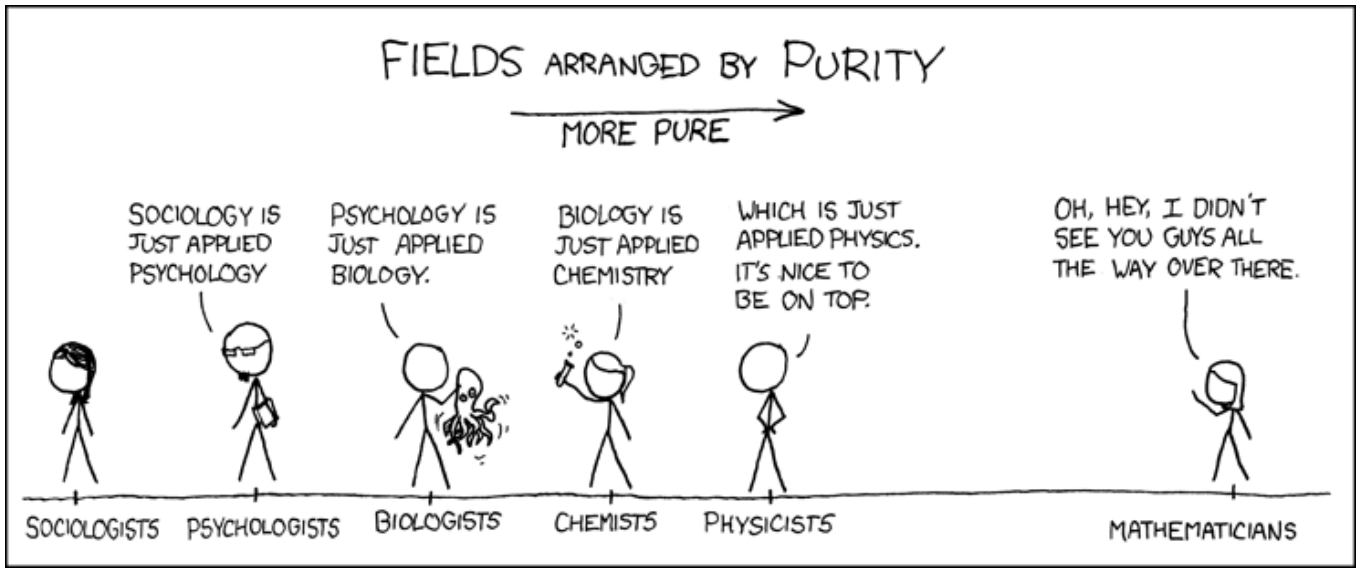
Projective Geometry: $\mathbb{R}P^2$





- Elegant
- Duality between points and lines
- Conics

Hierarchies of Geometries via Transformations



- Arthur Cayley: “projective geometry is all geometry”

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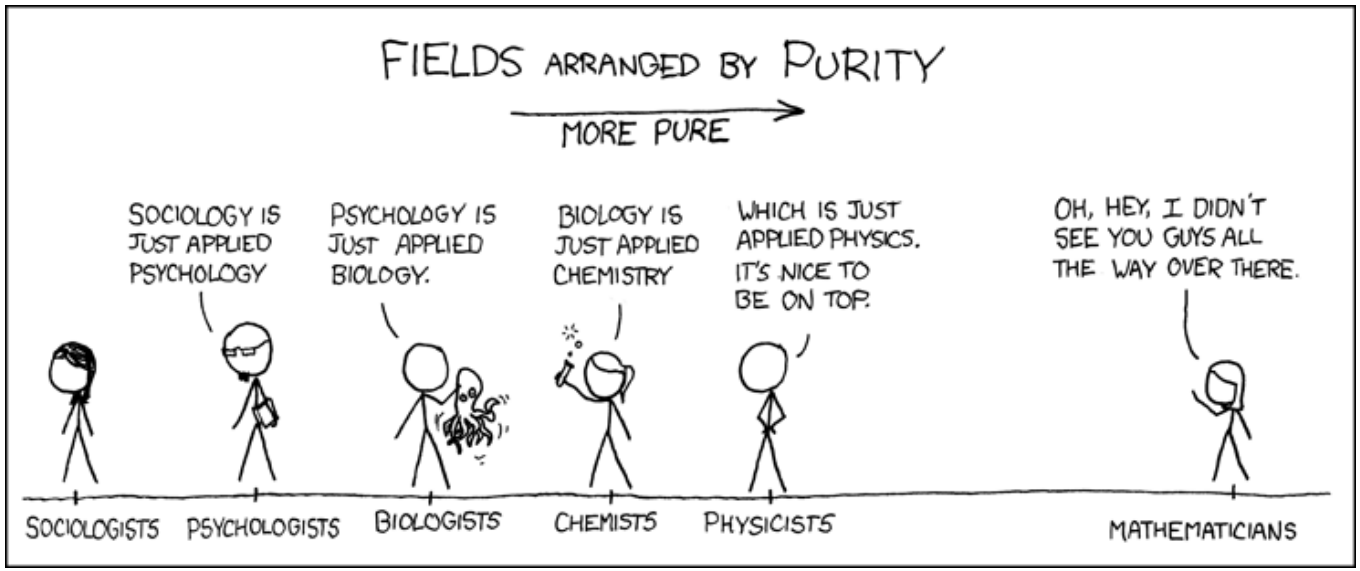

- Arthur Cayley: “projective geometry is all geometry”
- Euclidean transformations $\subset$ Similarity transformations (includes scalings) $\subset$ projective transformations
- Spherical and hyperbolic $\subset$ projective
- smaller the transformation group, the more rigid and more invariants.