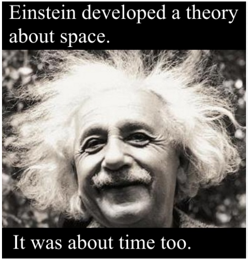
SpaceTime-Time: Special Relativity



Albert Einstein special relativity (1905)
Hermann Minkowski 4D spacetime model (1908)

$$\begin{bmatrix} t & x & y & z \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$$

Yardstick plus clock!

SpaceTime-Time: General Relativity (1915)

We've been following the 1916 paper: "The Foundation of the General Theory of Relativity"

- Einstein replaced the corollary we proved last class with

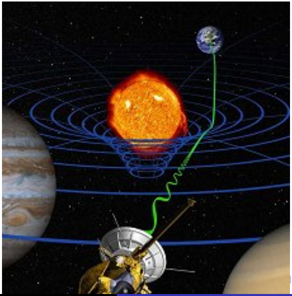
$$\frac{d^2x^\lambda}{ds^2} = -\Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}$$

- $\frac{\partial\varphi}{\partial x^i}$ & $\Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{ds}$ similar roles: Field equations relate these potential functions to the distribution of matter
- Field equations written in the Christoffel symbols:

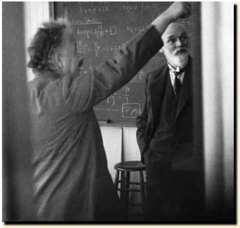
$$\frac{\partial\Gamma^\lambda_{\mu\lambda}}{\partial x^\nu} - \frac{\partial\Gamma^\lambda_{\mu\nu}}{\partial x^\lambda} + \Gamma^\beta_{\mu\lambda}\Gamma^\lambda_{\nu\beta} - \Gamma^\beta_{\mu\nu}\Gamma^\lambda_{\beta\lambda} = 0$$

Consequences and Experiments

- Spacetime in the presence of masses is curved and geodesics more interesting
- Gravity is the curvature of spacetime
- Arthur Eddington (1919): star near sun shifted by amount predicted by relativity! $\rightarrow$ Einstein public figure
- Radio sources
- Gravitational lensing and LIGO gravitational waves
- Precession of the orbit of Mercury



Solutions for General Relativity



- 2nd order PDE 16 eqs and 16 unknowns
- Einstein: “Cosmological Considerations in the General Theory of Relativity” (1917)

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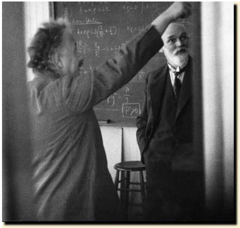

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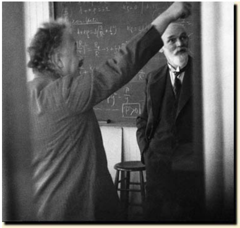
Solutions for General Relativity



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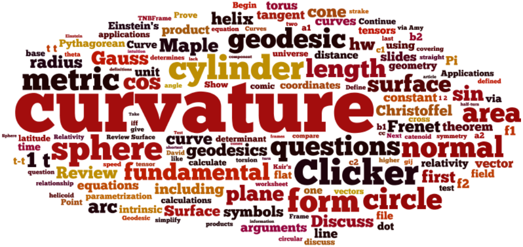
- de Sitter: $\mathbb{R} \times S^3$ (1917)

Course Overview

- Curves: torsion and curvature, piece together (DEs) to give us a nice curve. Frenet frame basis
- Surfaces: 1st and 2nd fundamental forms. 1st and Christoffel symbols intrinsic $\to$ Gauss curvature. E, F, G piece together to give us a nice smooth surface, or one with manageable singularities like the cone. l, m, n determine how the surface sits in space (if it does)
- SpaceTimes: g_{ij} and Γ^k_{ij} more terms $\to$ curvature tensors. Einstein's field equations: way to solve for "nice" solutions.



- Final Research Pres: review + extension + bib + peer review