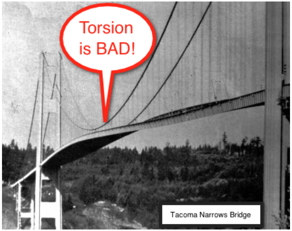
Solve for T, N, κ, B, τ by-hand

- $T(t) = \frac{\alpha'(t)}{|\alpha'(t)|}$ because $\frac{ds}{dt} = |\alpha'(t)|$ and $T = \frac{\alpha'(t)}{\frac{ds}{dt}}$
- $\vec{\kappa} = T'(s) = \frac{T'(t)}{|\alpha'(t)|}$ because $\vec{\kappa} = \frac{dT}{ds} = \frac{dT}{dt}\frac{dt}{ds} = \frac{\frac{dT}{dt}}{\frac{ds}{dt}} = \frac{T'(t)}{|\alpha'(t)|}$
- curvature κ: length of $\vec{\kappa} = |\vec{\kappa}|$ and $N(t) = \frac{\vec{\kappa}}{|\vec{\kappa}|}$
- $B(t) = T \times N$
- torsion τ: compute $\frac{B'(t)}{|\alpha'(t)|}$ & compare it to N (they are multiples of each other) to find $-\tau$ and then τ because $\frac{B'(t)}{|\alpha'(t)|} = \frac{B'(t)}{\frac{ds}{dt}} = \frac{dB}{dt}\frac{dt}{ds} = B'(s)$ (by chain rule), and we defined this as $-\tau N$.

$\alpha(t) = (\frac{5}{13}\cos(t), -\sin(t), -\frac{12}{13}\cos(t))$

T, κ and N work in higher dimensions, but the osculating plane is not defined by a normal, nor does cross product make sense—that is replaced by tensors and forms.



http://pedemmorsels.com/wp-content/uploads/2014/01/Torsion.jpg

- First show that B' has no B component

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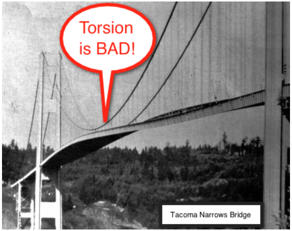

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- First show that B' has no B component
- So $B' = aT + bN + 0B$. Next show B' has no tangential component via a cross product argument, using $B = T \times N$

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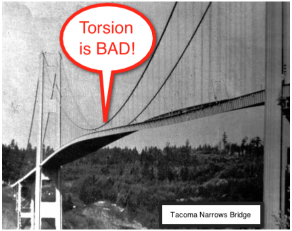

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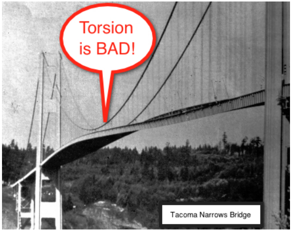

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- First show that B' has no B component
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$$B' = T' \times N + T \times N'$$
$$= \kappa N \times N + T \times N'$$
$$= \kappa |N||N| \sin 0 + T \times N' = 0 + T \times N'$$

So B' is perpendicular to T too and thus has only an N component—we define the component to be $-\tau$

TNB Derivatives

We defined $T' = \kappa N$

We showed B' had no B component and no T component and thus it makes sense to define $B' = -\tau N$.

Assume these relationships above.

- Show that N' has no N component
- Show that N' has a $-\kappa$ component of T and a τ component of B

We defined $T' = \kappa N$

TNB Derivatives

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Assume these relationships above.

- Show that N' has no N component
- Show that N' has a $-\kappa$ component of T and a τ component of B
  $T \cdot N = 0$ so $T' \cdot N + T \cdot N' = 0$ and
  $T \cdot N' = -T' \cdot N =$

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  $T \cdot N' = -T' \cdot N = -\kappa N \cdot N = -\kappa$
  $T \cdot N' = T \cdot (aT + bN + cB) = aT \cdot T + bT \cdot N + cT \cdot B = a$
  so $a = -\kappa$.
  Similarly,

We defined $T' = \kappa N$

TNB Derivatives

We showed B' had no B component and no T component and thus it makes sense to define $B' = -\tau N$.
Assume these relationships above.

- Show that N' has no N component
- Show that N' has a $-\kappa$ component of T and a τ component of B
  $T \cdot N = 0$ so $T' \cdot N + T \cdot N' = 0$ and
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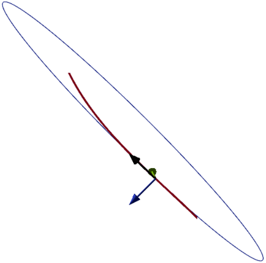
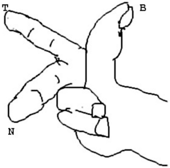
  so $a = -\kappa$.
  Similarly,
  $B \cdot N = 0$ so $B' \cdot N + B \cdot N' = 0$ and
  $B \cdot N' = -B' \cdot N = - - \tau N \cdot N = \tau$
  so $b = \tau$ in $aT + bN + cB$
  $N' = -\kappa T + 0N + \tau B$ QED

TNB Derivatives





http://www.rudyrucker.com/transrealbooks/collectedessays/images/kaptauhand.jpg

$$T'(s) = \kappa N$$
$$N'(s) = -\kappa T + \tau B$$
$$B'(s) = -\tau N$$

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

Warehouse 13's main characters Myka and Pete are trapped in a lemniscate in "The Greatest Gift." [Syfy, Universal Studios]

https://drive.google.com/file/d/18mfbulz3AgBwEuNYTo0XZJ8At_BYwp5v/view?usp=sharing

Warehouse 13's main characters Myka and Pete are trapped in a lemniscate in "The Greatest Gift." [Syfy, Universal Studios]

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a lemniscate can be parameterized so that the metric does expand [Amy Ksir, –]

- Lemniscate of Bernoulli $(\frac{3\cos t}{1+\sin^2 t}, \frac{3\sin t\cos t}{1+\sin^2 t}, 0)$
- Lemniscate of Myka $(\frac{t+t^3}{1+t^4}, \frac{t-t^3}{1+t^4}, 0)$