Arc Length, T, Velocity, Speed, Acceleration, Jerk

Dr. Sarah's Differential Geometry

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Try to help each other! Discuss and keep track of any questions your group has. Feel free to ask me questions during group work time as I make my way around as well as when I bring us back together.

- 1. Sit in a group of 4 (if possible) and introduce yourselves to those sitting near you. What are their preferred first names?
- 2. Which of the following is $\alpha'(s)$ where s is the arc length parameter?
 - a) velocity
 - b) unit tangent vector T
 - c) curvature
 - d) more than one answer works
 - e) none of the above

Answer on pollev and prepare to share from your group's discussion. This may take the form of an assertion, question, definition, example, or other connection. It could also be something you tried and rejected. There may be a lag at times—review related concepts and examples, add to your notes, or get to know each other!

- 3. How did chain rule arise in the arc length s, T, velocity, speed, acceleration and jerk interactive video? Answer on pollev and review with your group.
 - a) In the prior video on the tractrix, it was a part of the computation of the arc length of the tractrix as it was needed for the velocity and hence speed, and we used that again in this video
 - b) It arose in the proof that every differentiable curve that is regular can be reparameterized by arc length
 - c) When we are computing T(t) instead of T(s), it's chain rule at work!
 - d) all of the above
 - e) exactly two of the above
- 4. Work with neighbors or check-in with them regularly on the general helix $\alpha(t) = (a \cos(t), a \sin(t), bt)$ where $a, b \in \mathbb{R}$ constants
 - Compute unit tangent $T(t) = \frac{\alpha'(t)}{|\alpha'(t)|}$ (leave a, b general)

• Compute arc length
$$s(t) = \int_0^t |\alpha'(u)| du$$

- Write the inverse function t(s) by solving for t
- Reparameterize the curve by arc length $\beta(s) = \alpha(t(s))$



- http://previews.123rf.com/images/limbi007/limbi0071302/limbi007130200034/17726502-Orange-cartoon-characters-runs-on-the-green-helix--Stock-Photo-orangejpg
- 5. If you are finished before I bring us back together and you have access, consider the differential geometry of the helix in Maple by modifying the project 1 Maple file that is accessible from the project 1 activities in ASULearn as follows:

Be sure to execute the packages by hitting return in the with(Student[VectorCalculus]): with(plots): line. Then enter and execute each of these:

x:=a*cos(t); y:=a*sin(t); z:=b*t;

Next, continue executing commands to look at Maple's output for velocity, acceleration, jerk, speed, arc length and the first TNBFrame command. Maple won't plot with generic a and b so next skip down to the curvature and torsion commands.

- 6. Redo the Maple with specific values up top by replacing a with 3 and b with 4.
- 7. Discuss the following:



In physics and geometry, the lolcatenary is the curve that an idealized hanging lolcat assumes under its own weight when supported only at its ends.

8. If finished before I bring us back together, review from the video due today—you can access video slides from the "in-class items, video slides and more" page at the top of ASULearn—or look at or discuss upcoming work like project 1.