Curvature

Dr. Sarah's Differential Geometry

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Try to help each other! Discuss and keep track of any questions your group has. Feel free to ask me questions during group work time as I make my way around as well as when I bring us back together.

1. Sit in a group of 4 (if possible) and introduce yourselves to those sitting near you. What are their preferred first names?

2. If you have technology with you, like a phone, tablet, or computer, then answer the poll question on the front slide via responding at
   `https://pollev.com/drsarah314`
   If not, then answer in your notes.

3. Various curvatures will be important all semester long.

   (a) In Calculus with Analytic Geometry III, you may have seen the curvature of a plane curve $y = f(x)$ at the point x_0 as the scalar $\kappa = \dfrac{f''(x_0)}{(1+f'(x_0)^2)^{\frac{3}{2}}}$. Compute this by-hand for $y = x^2$ and compare with your group.

   (b) Evaluate κ at $x_0 = 0$.

   (c) The tangent to a curve is the best fitting line. The radius of the best fitting circle, called the osculating circle, is $\dfrac{1}{\kappa}$, so what is the radius of the best fitting circle at $x_0 = 0$?

   (d) Using standard mathematical axes with x as horizontal and y as vertical, sketch the curve $y = x^2$ by-hand.

   (e) Roughly add to your curve sketch what seems to you to be the best fitting circle that hugs the curve at $x_0 = 0$ and compare with your group.

   (f) Does the best fitting circle have the same radius at other points? Discuss with your group and then write in your notes why or why not.

   (g) What happens to the curvature when $y = mx + b$ instead of $y = x^2$?

4. Discuss with each other as you review content from the lines and Maple interactive video including, one at a time:

   1) $\alpha(t)$ is a curve that is a constant speed straight line iff the acceleration is $\vec{0}$.

   2) Why a line $l(t) = \vec{p} + t(\vec{q} - \vec{p})$ is shorter than any other curve $\alpha(t)$ between $\vec{p}$ and $\vec{q}$ in Euclidean geometry.

   3) Maple intro related to the cardioid and lines.

   What are significant takeaways of each? Also reflect on personal connections and/or any remaining questions you have. Each group member takes a turn for each. Try to help each other solidify and review! I want to remind you that you can access slides for the video at the top of ASULearn from the in-class items, video slides and more link.

   Then select a board to write or sketch a significant takeaway—try to add items other groups don't already have up.

5. If you are finished before we come back together, first ensure that your entire group is finished too, and if not, help each other. Then continue reviewing from the video or prior worksheet or look at or discuss upcoming work.